Reinforcement Learning the Learning Rate in Probabilistic Reversal Learning Tasks

Semester Project

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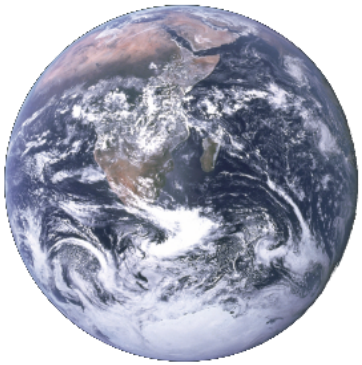
Supervisor: Dr. Kerstin Preuschoff

Department of Economics, University of Zürich

May 25, 2011

Outline

Motivation and Question



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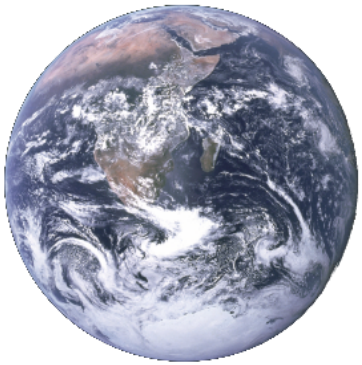

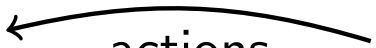

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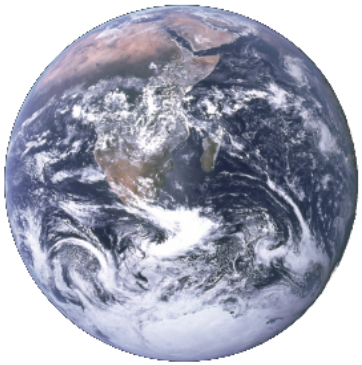
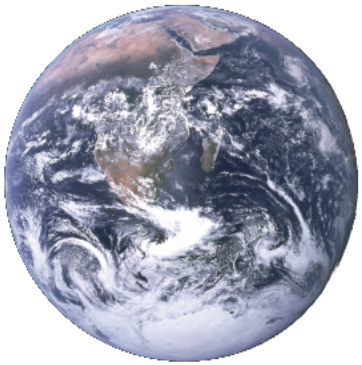


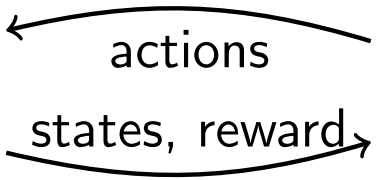
actions

states, reward

Motivation and Question

- Truth* changes
  $\longrightarrow$ Volatility





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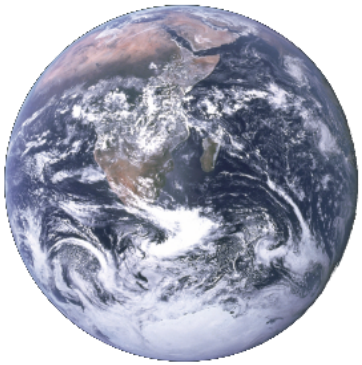
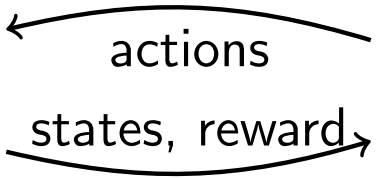
Motivation and Question





- Truth* changes
  $\longrightarrow$ Volatility
- Observation* is noisy
  $\longrightarrow$ Risk

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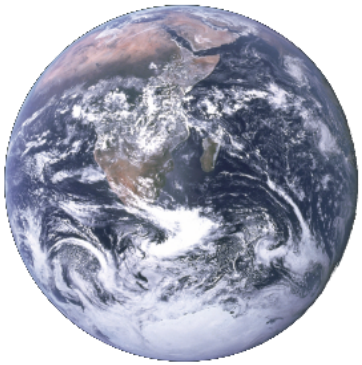
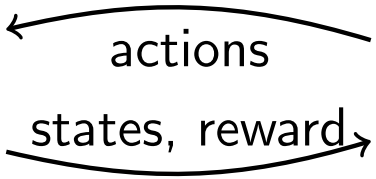
Motivation and Question





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- How conservative do I change my estimate of the truth?
  $\longrightarrow$ Learning rate λ

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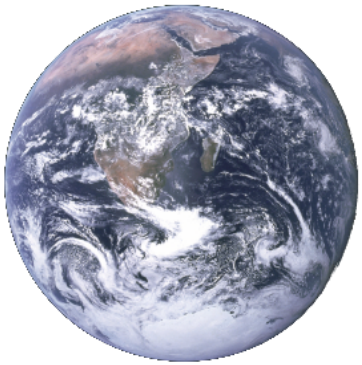

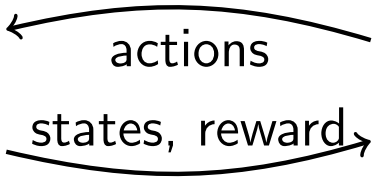

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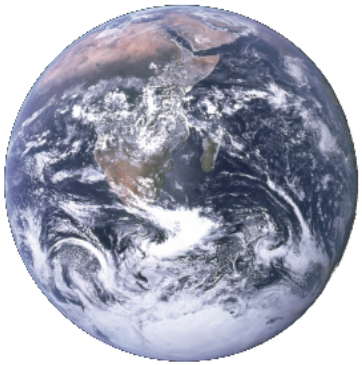

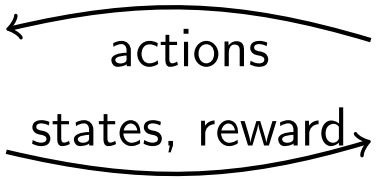

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Task Explore the suggested algorithm for different parameters and compare to other methods. This involves parameters for the Q-learning as for the tabu policy learning method.

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Probabilistic Reversal Learning

Real Experiment

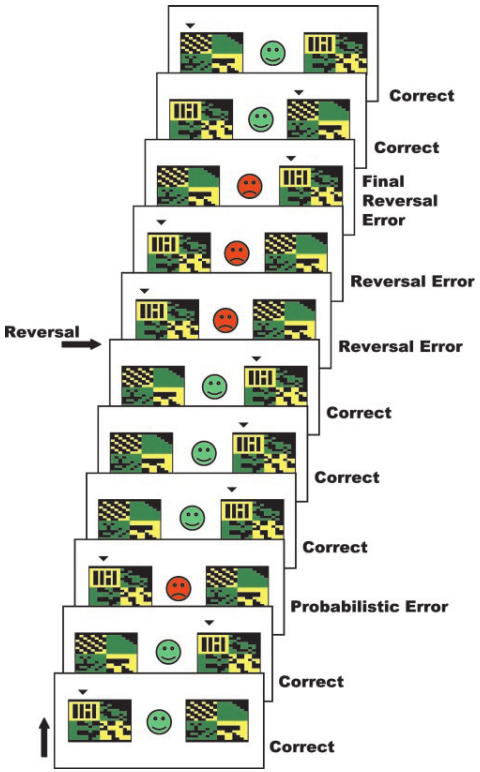

Figure: Cools et al. 2002

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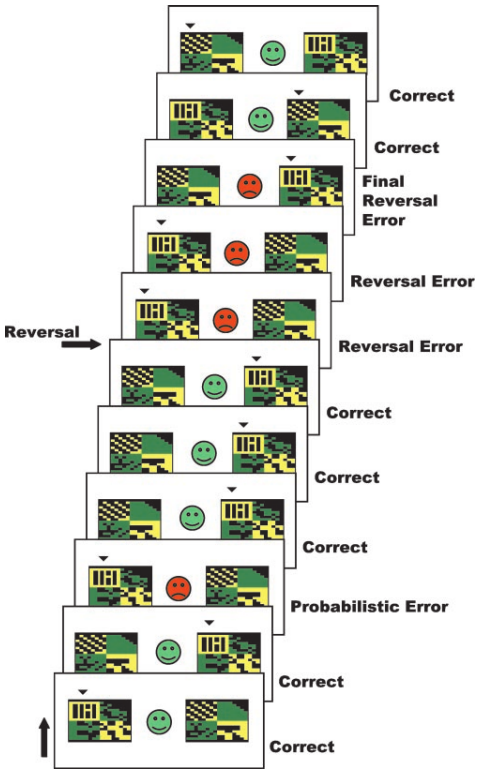

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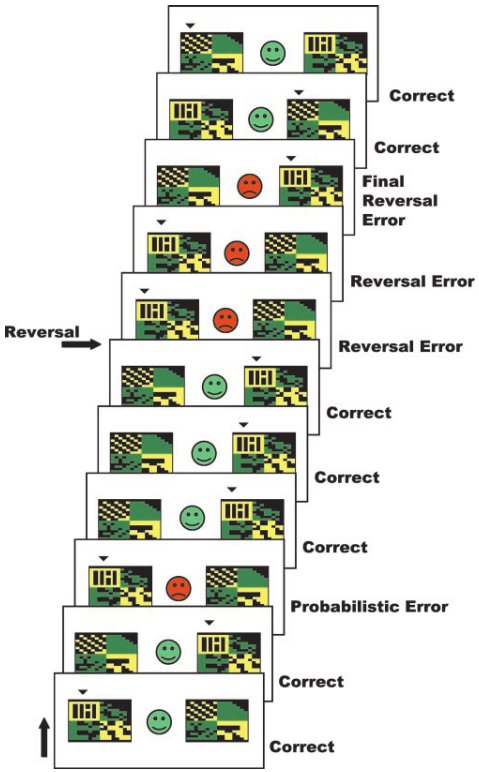

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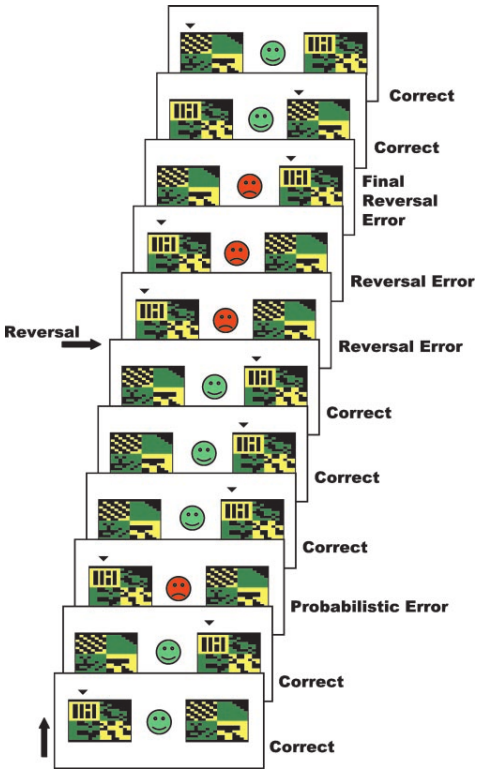
Model

Env Sets the better stimuli $u_t \in \{r, l\}$

Probabilistic Reversal Learning

Real Experiment



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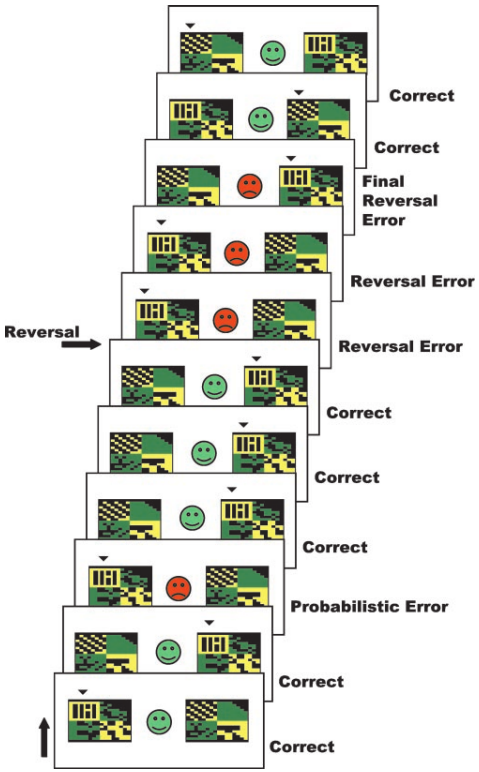

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$$\Pr\{\text{reward} \mid \mathbf{a}_t = u_t\} = p$$
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Probabilistic Reversal Learning

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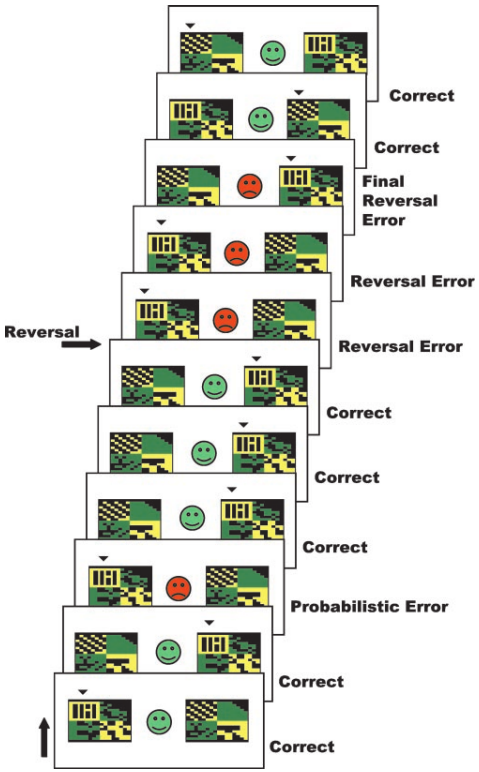

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action	reward	better stimuli $\mathbf{y}_t$
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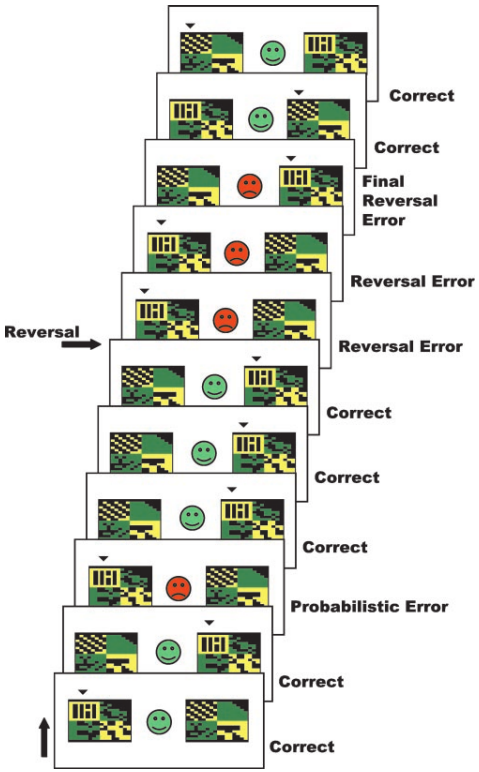

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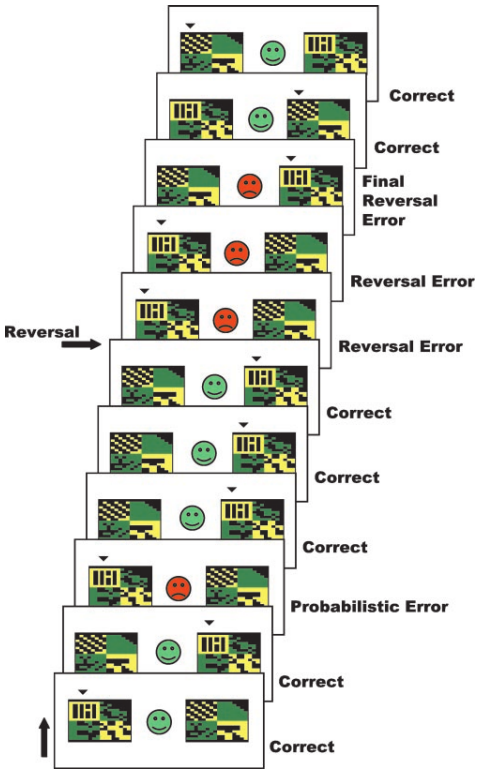

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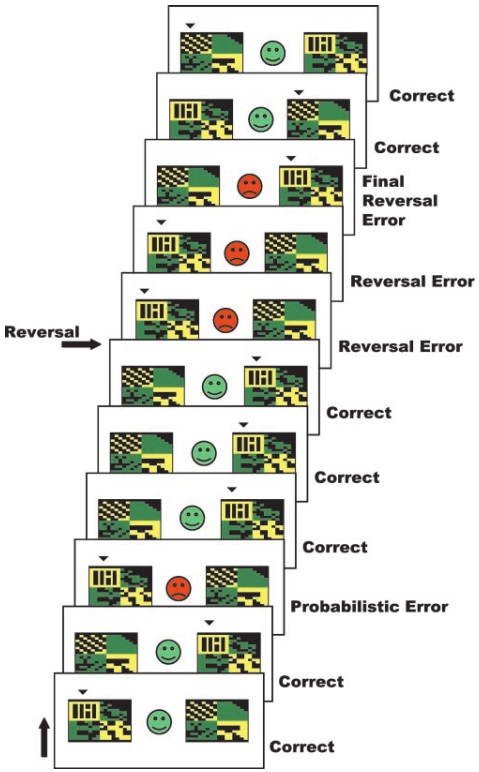

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Our Experiment

1. code left/right $\longrightarrow 1/0$
2. kick the action-reward thing*

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Probabilistic Reversal Learning

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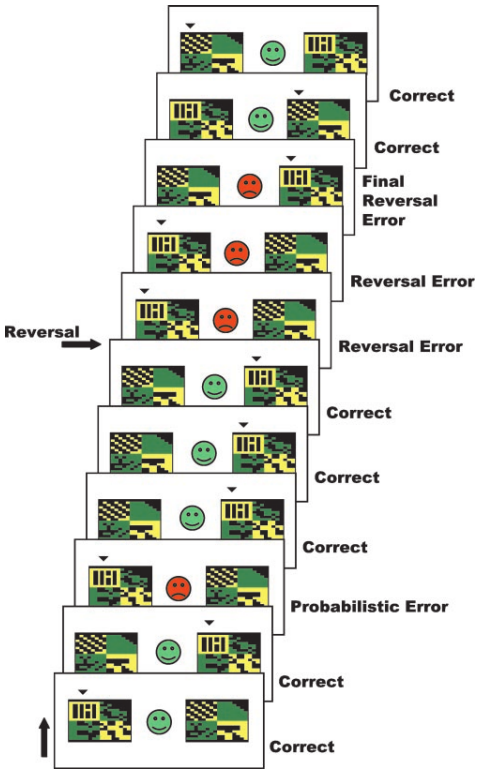

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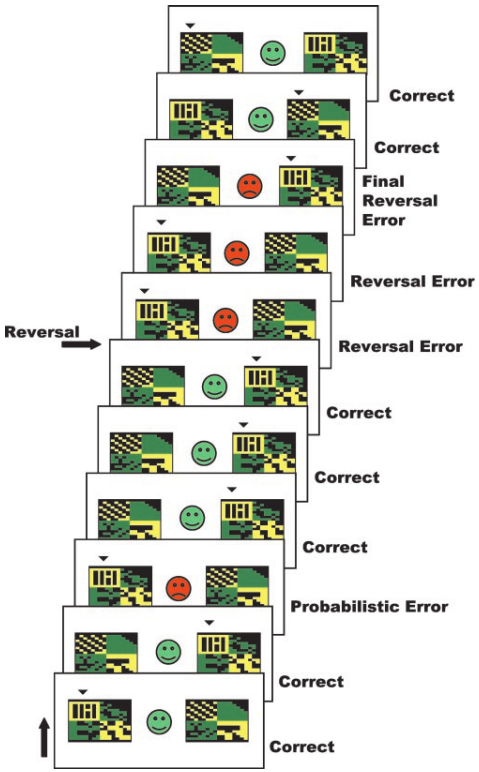

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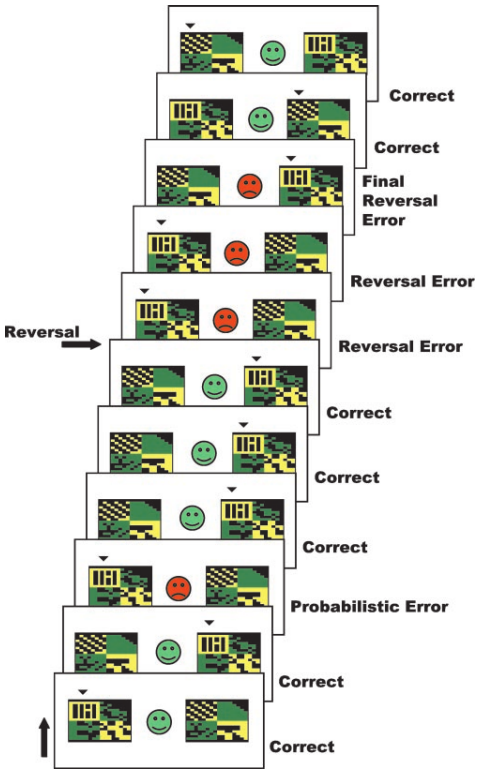

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Env Reveals

$$\mathbf{y}_t \sim \text{Bern}(x_t)$$

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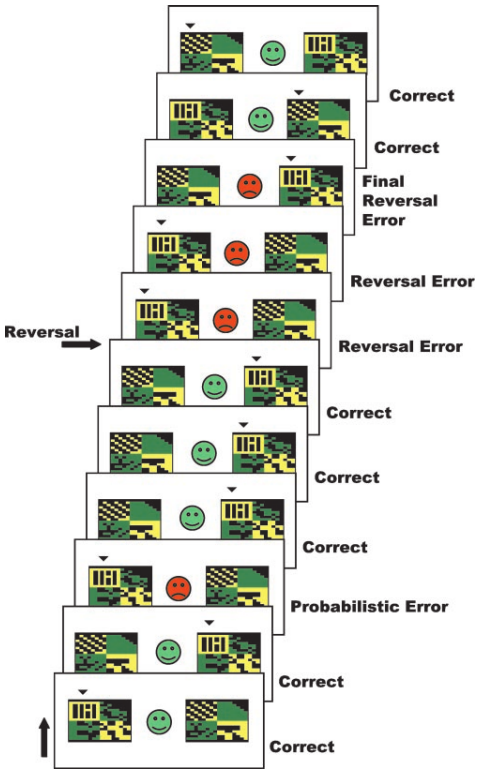

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Ag Estimates x_t by discounted recursive mean (ML):

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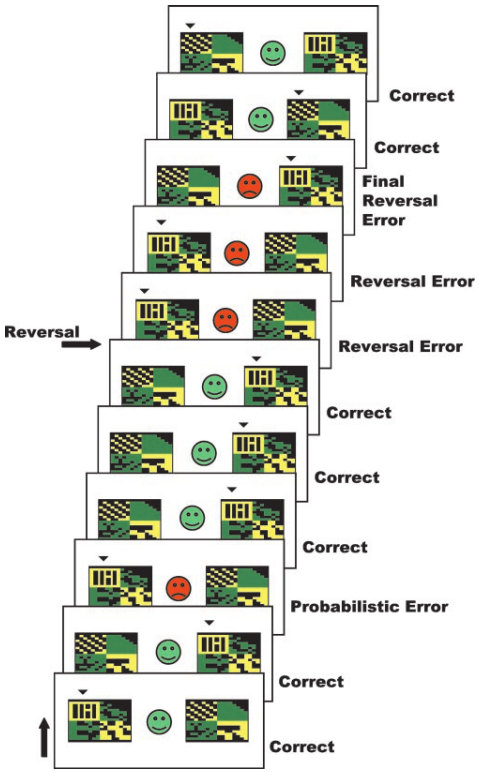

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Env Generate

$$x_t = \begin{cases} p & \text{if } u_t = 1 \\ 1 - p & \text{if } u_t = 0 \end{cases}$$

Env Reveals

$$\mathbf{y}_t \sim \text{Bern}(x_t)$$

Ag Estimates x_t by discounted recursive mean (ML):

$$\hat{\mathbf{x}}_t = \hat{\mathbf{x}}_{t-1} + \lambda \left(\mathbf{y}_t - \hat{\mathbf{x}}_{t-1}\right)$$

Ag Estimates u_t:

$$\hat{\mathbf{u}}_t = \begin{cases} l & \text{if } \hat{\mathbf{x}} \geq 0.5 \\ r & \text{else} \end{cases}$$

* Instead of having a two complementary action model, we reduce it to a one action model. The action is given and $\mathbf{y}_t$ can be considered in a reinforcement learning context as the reward.

Reinforcement Learning

The Agent - Environment Setup

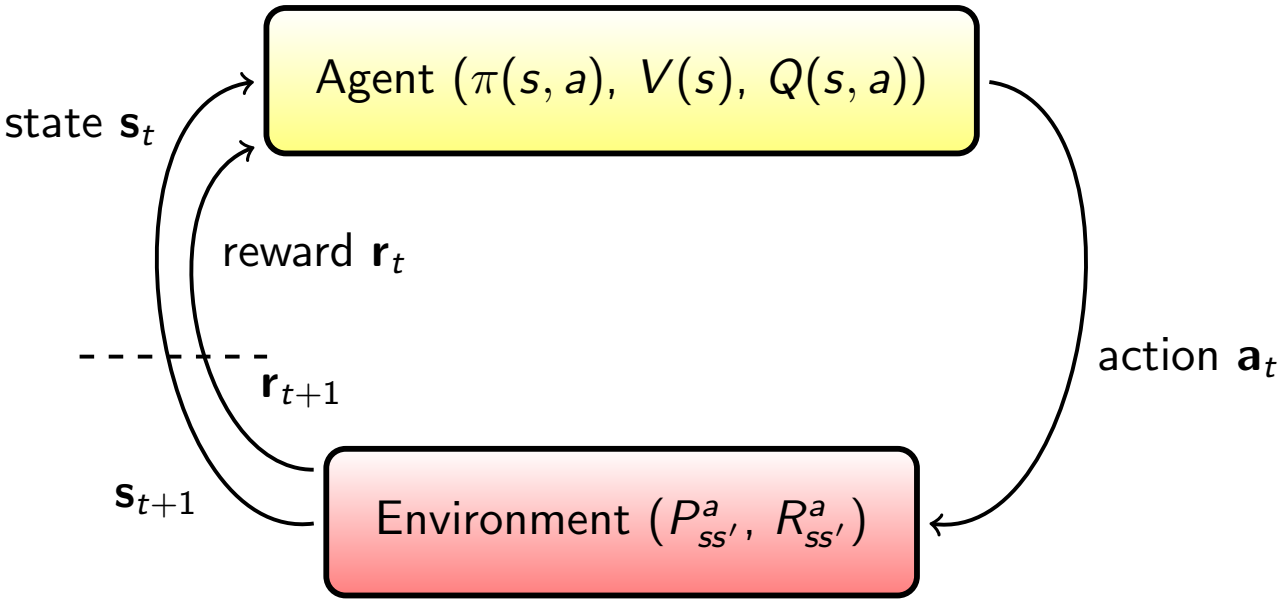
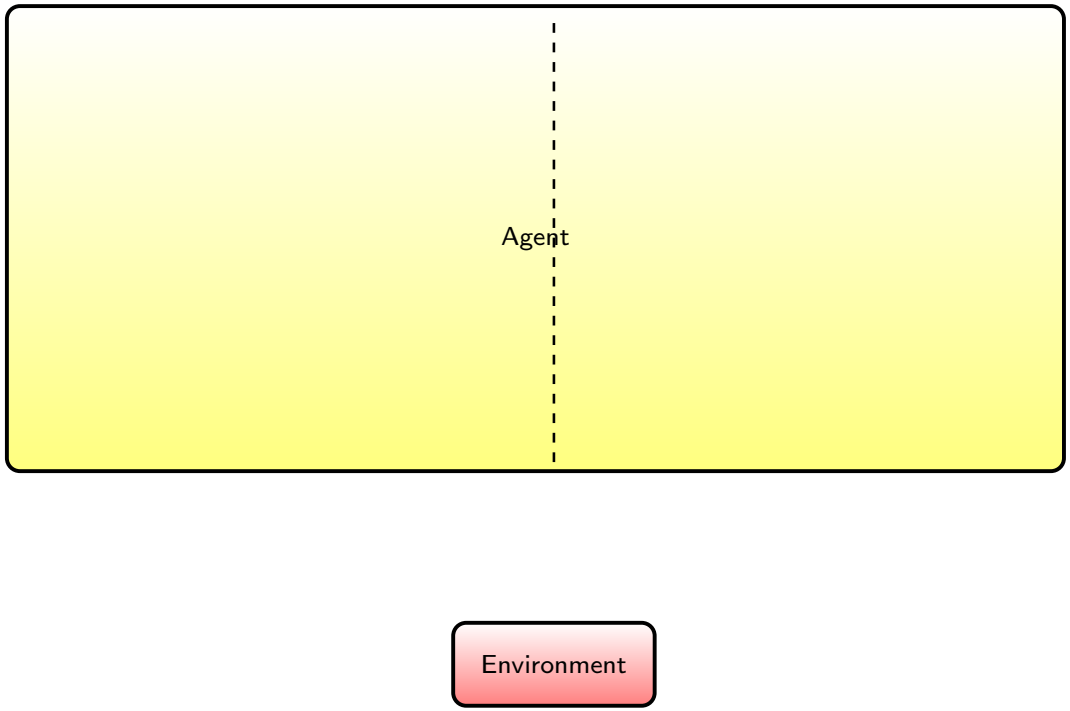


Figure: Reinforcement Learning Setup

Reinforcement Learning

Q-Learning



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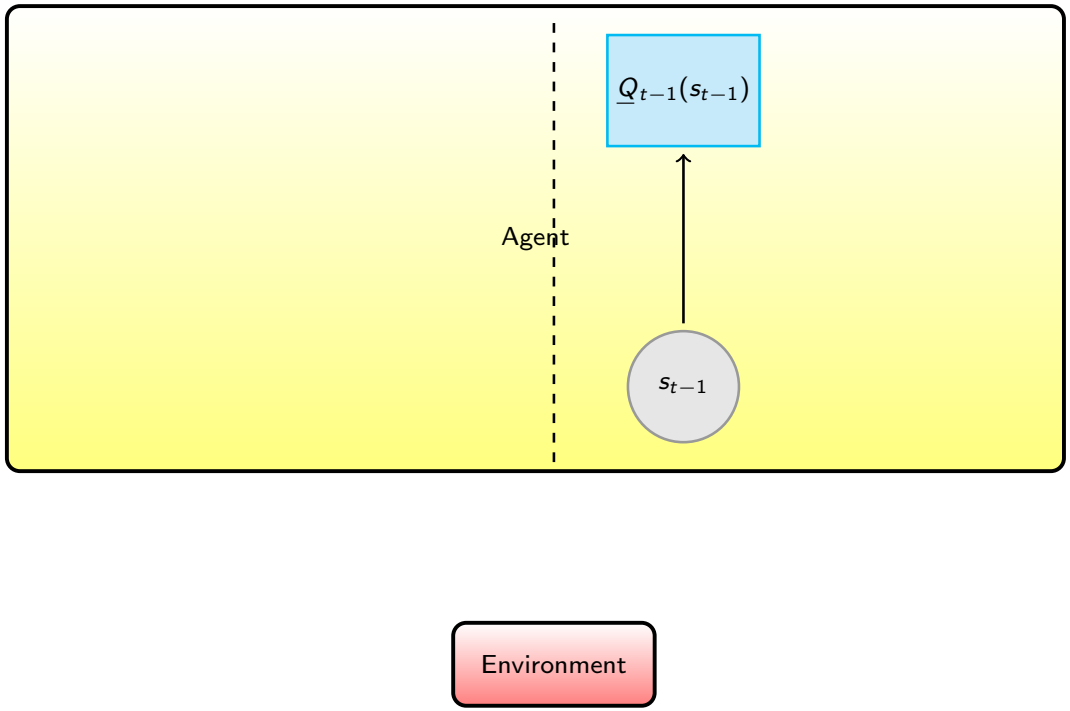
Reinforcement Learning

Q-Learning



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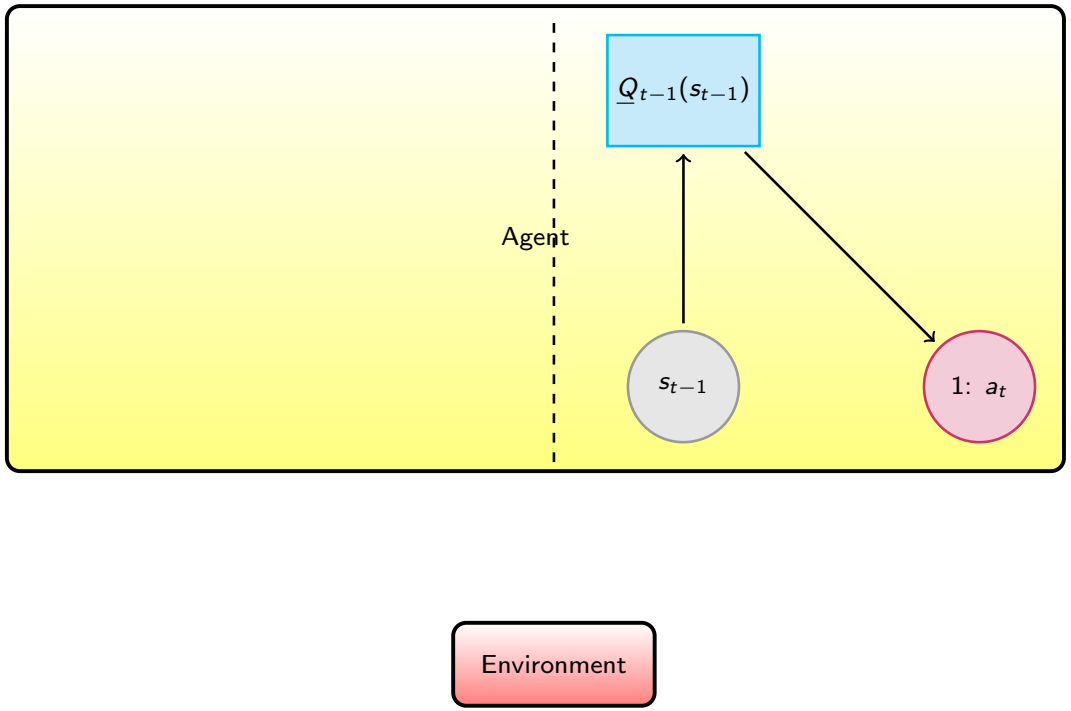

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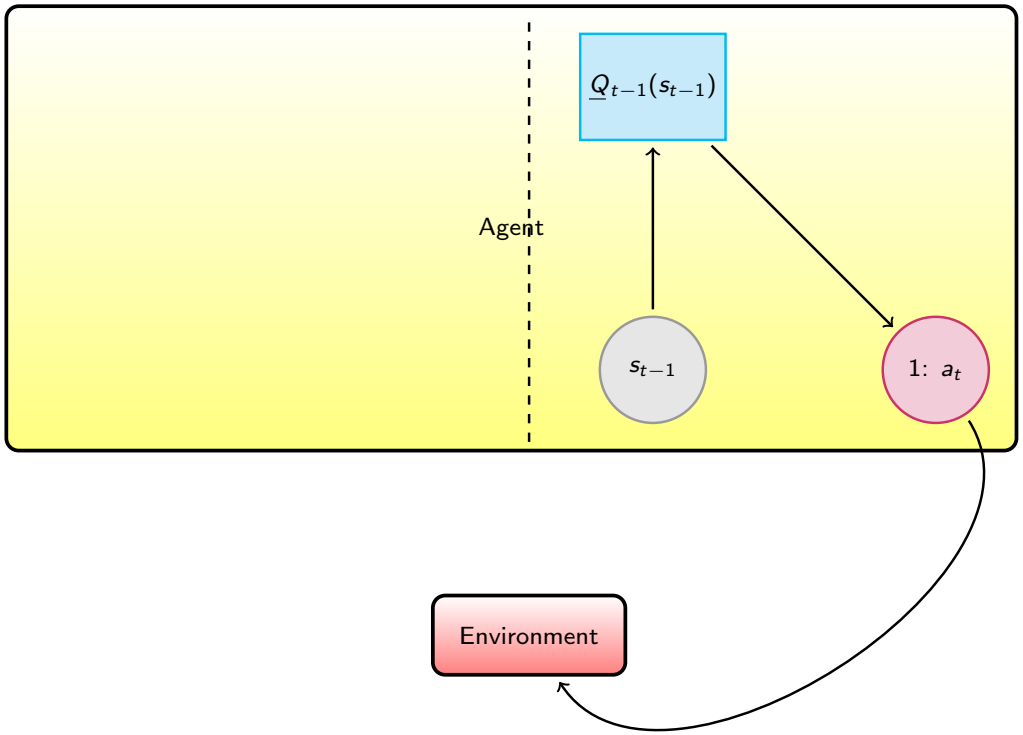

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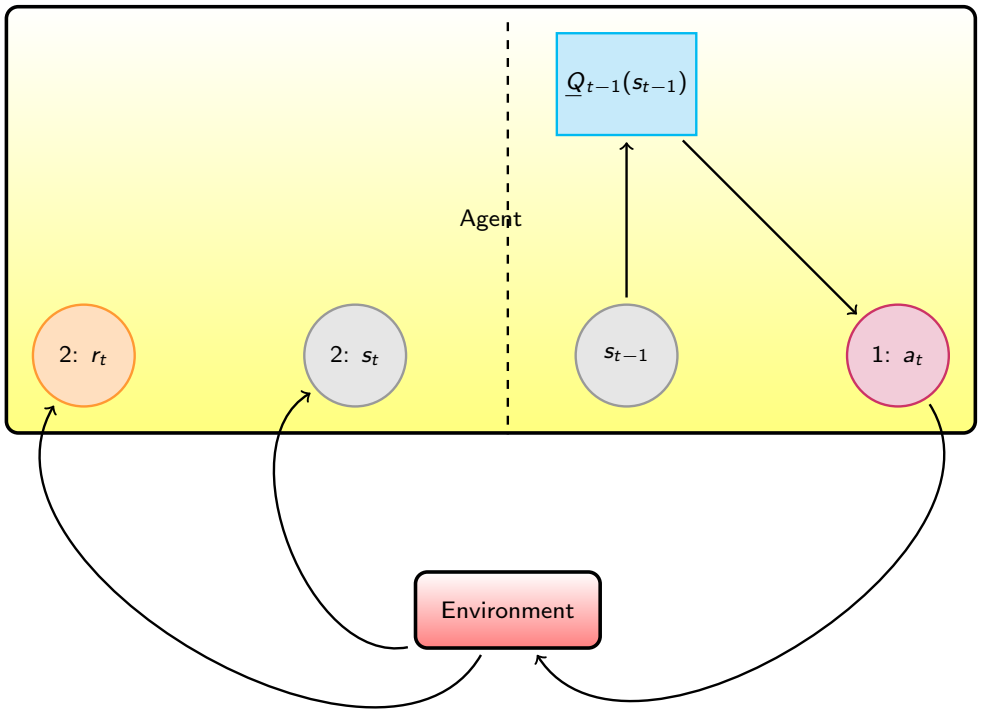

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Reinforcement Learning

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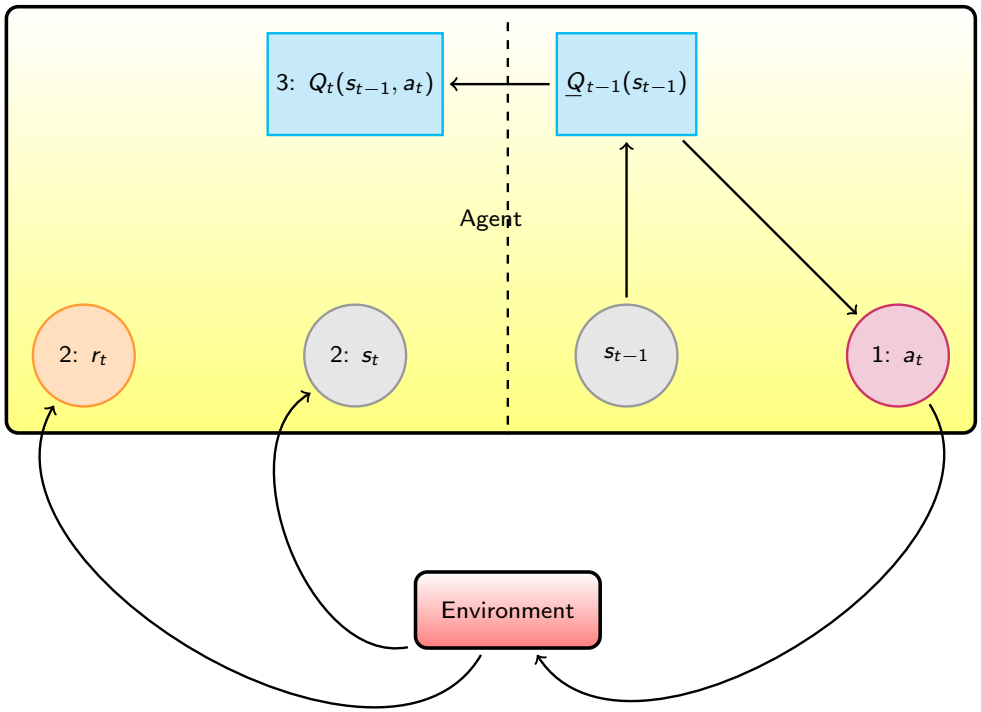

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$$Q_t \qquad = Q_{t-1} \qquad + \lambda \Big(\qquad - Q_{t-1} \qquad \Big)$$

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Reinforcement Learning

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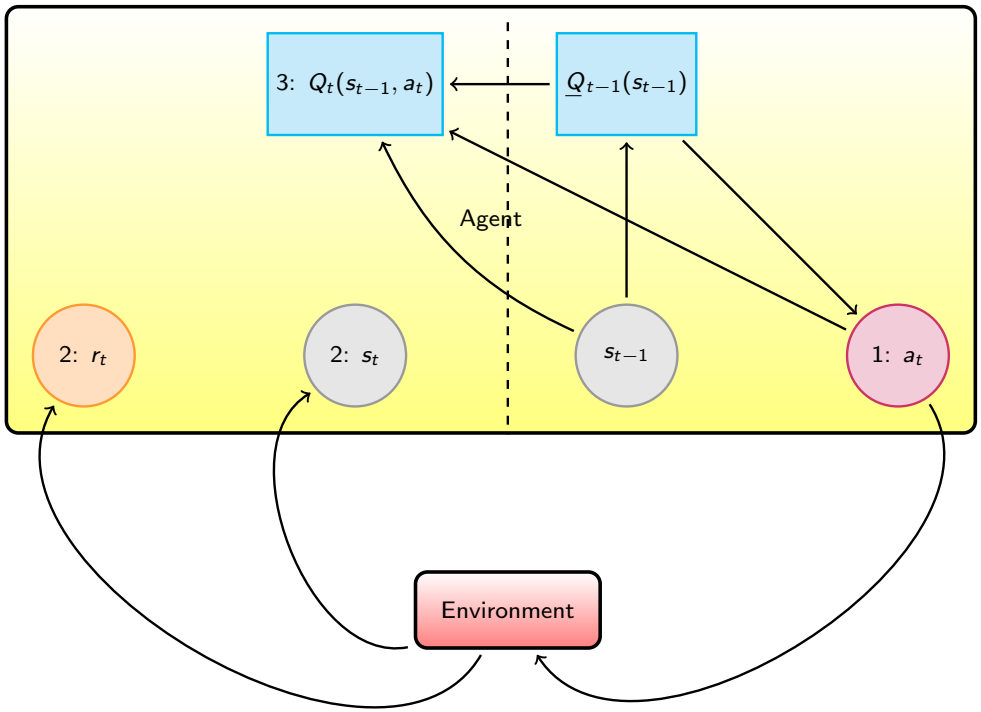

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Reinforcement Learning

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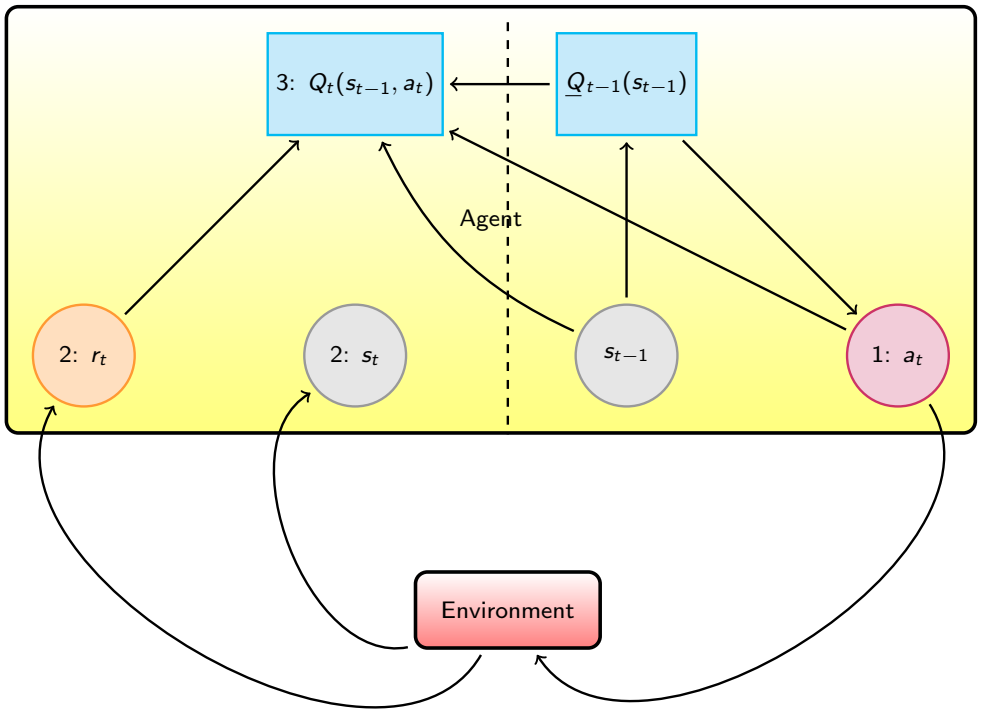

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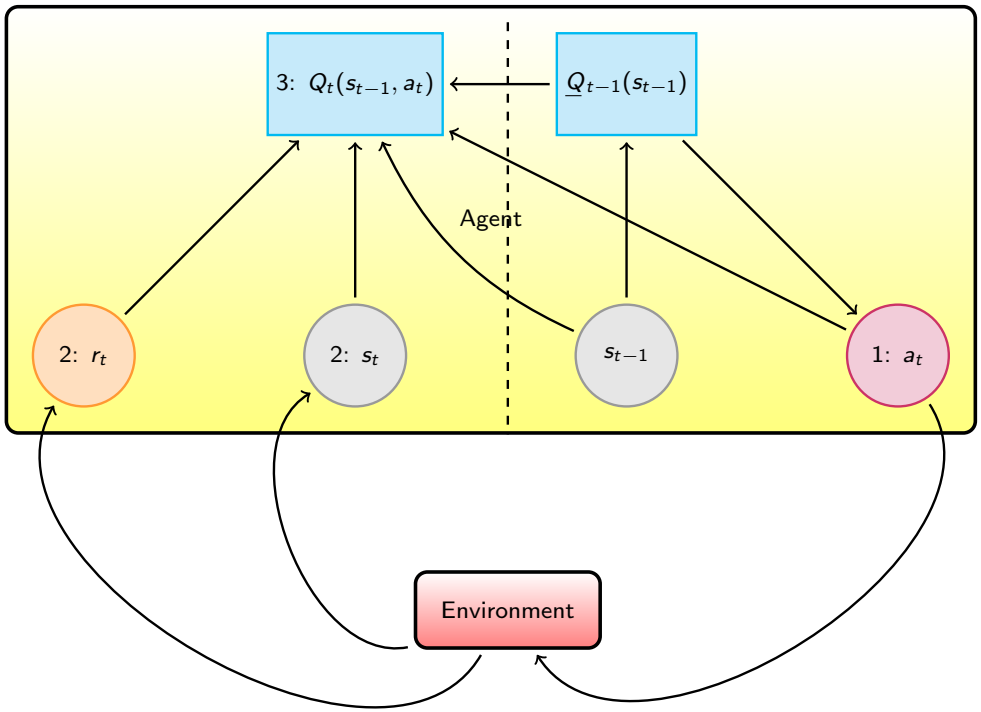
**Discounted future reward

Reinforcement Learning

Q-Learning



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**Discounted future reward

Q-Learning the Learning Rate

Overview

In General

- A first reinforcement learner solves the actual task (Agent 1).

Q-Learning the Learning Rate

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In Our Case (Probabilistic Reversal Learning)

- Agent 1 is a discounted recursive mean learner:

$$\hat{x}_t = \hat{x}_{t-1} + \lambda_t \underbrace{(y_t - \hat{x}_{t-1})}_{\eta_t},$$

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Q-Learning the Learning Rate

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- Agent 2 gets as a feedback (reward) η_t^2

$$Q_t(a_{t-1}) = Q_{t-1}(a_{t-1}) + \lambda 2 \left(\eta_t^2 + \gamma 2 \min \underline{Q}_{t-1} - Q_{t-1}(a_{t-1}) \right)$$

  - The set of actions are the possible learning rates for Agent 1.
  - It is a single state learner.
  - The default policy is a Tabu search policy.

Q-Learning the Learning Rate

Implementation

- Form of the RLPs: **action** $\longrightarrow$ **reward** $\longrightarrow$ **update**.

Q-Learning the Learning Rate

Implementation

- Form of the RLPs: **action** $\longrightarrow$ **reward** $\longrightarrow$ **update**.
- **Process 1**

Q-Learning the Learning Rate

Implementation

- Form of the RLPs: **action** → **reward** → **update**.
- **Process 1** and **Process 2**

Q-Learning the Learning Rate

Implementation

- Form of the RLPs: **action** $\longrightarrow$ **reward** $\longrightarrow$ **update**.
- **Process 1** and **Process 2** interact by r_t (u_t) and $u_t(a_t)$.

Q-Learning the Learning Rate

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At time step t

Q-Learning the Learning Rate

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At time step t

a_t -

Q-Learning the Learning Rate

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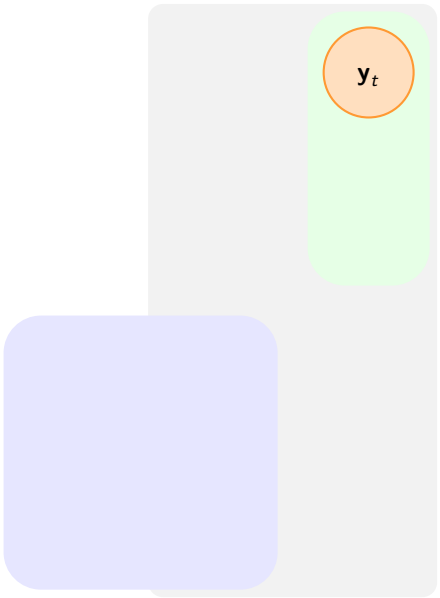

At time step t

a_t -

r_t It observes y_t

Q-Learning the Learning Rate

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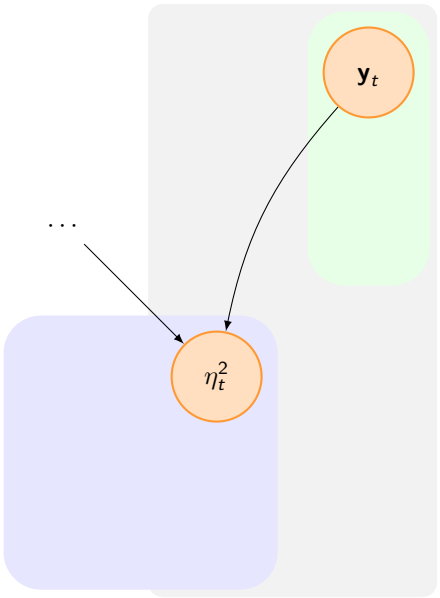

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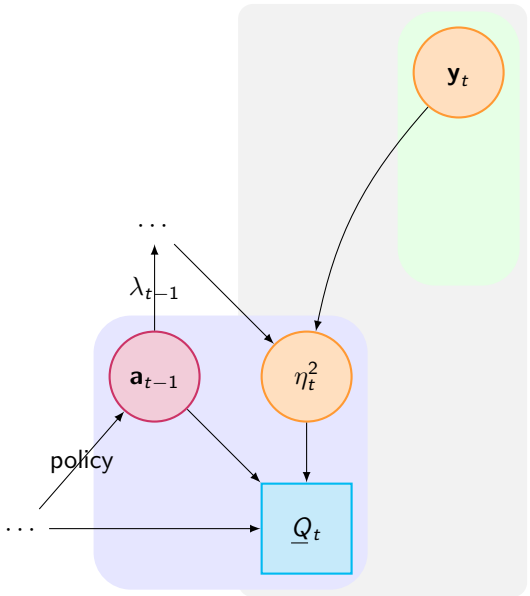

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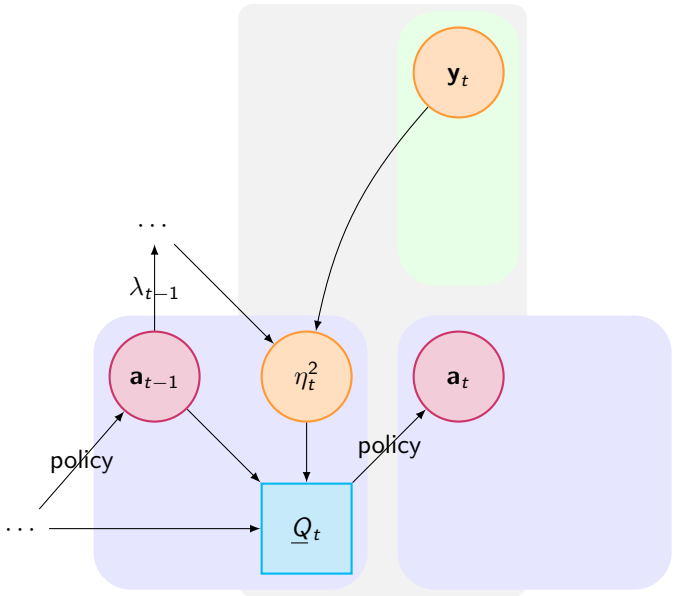

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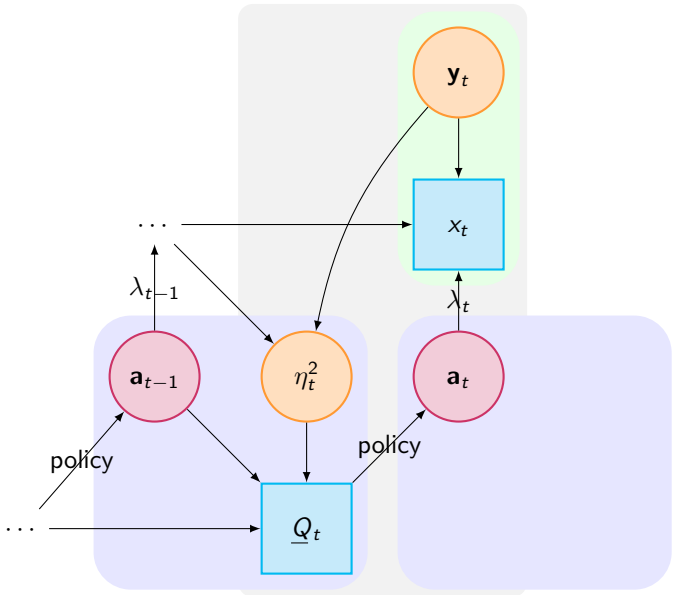

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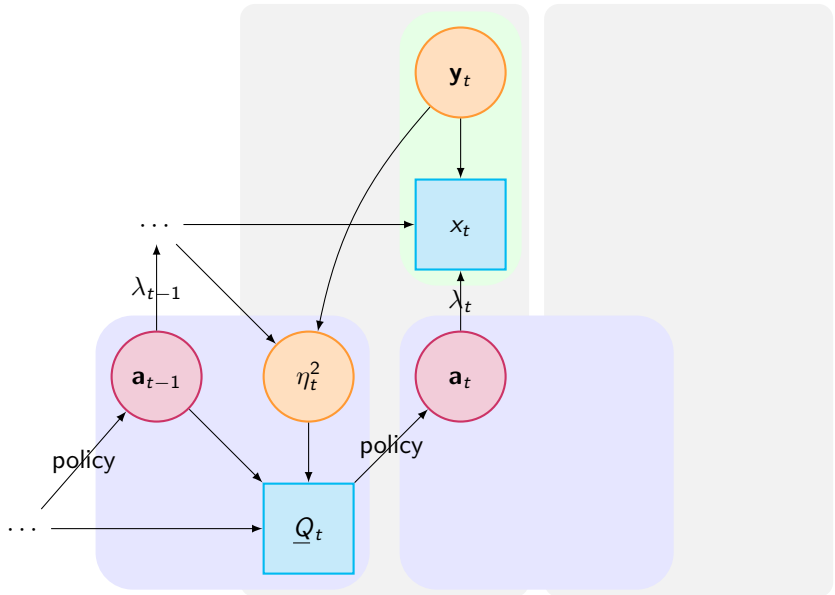

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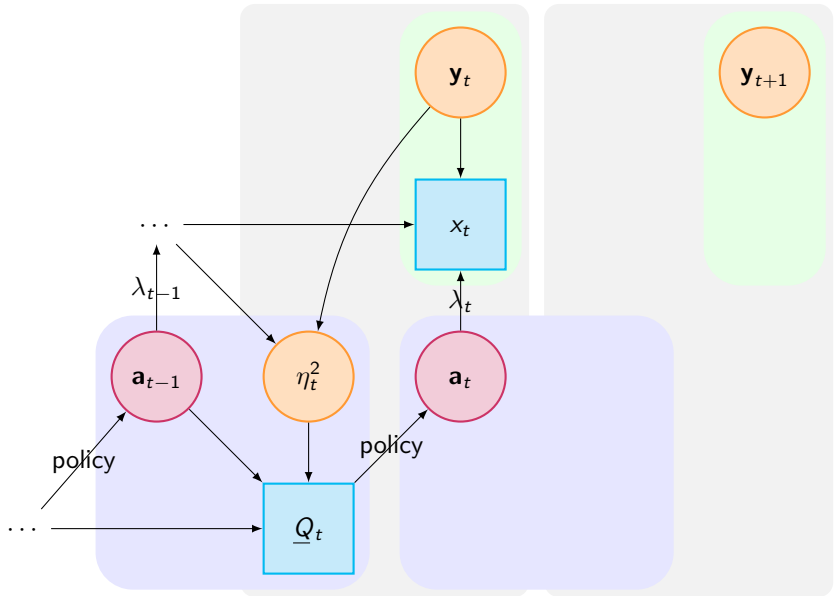

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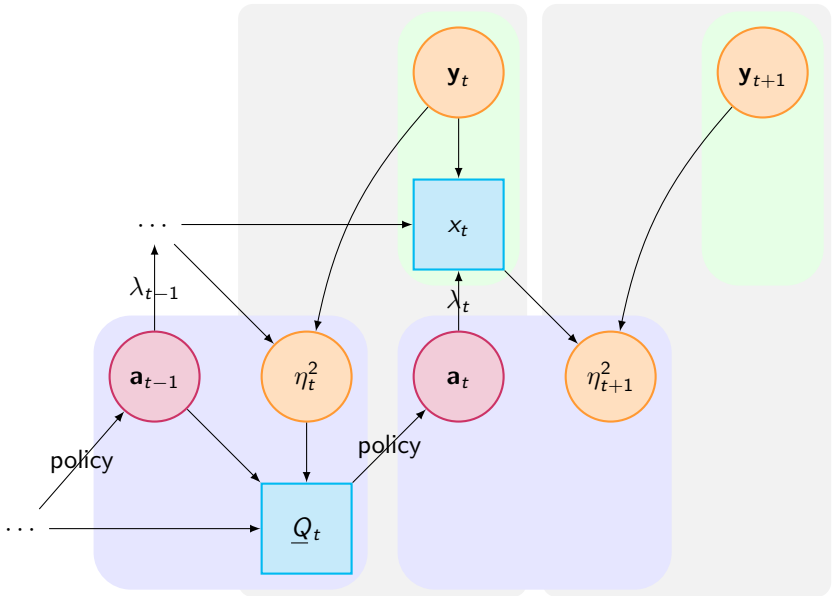

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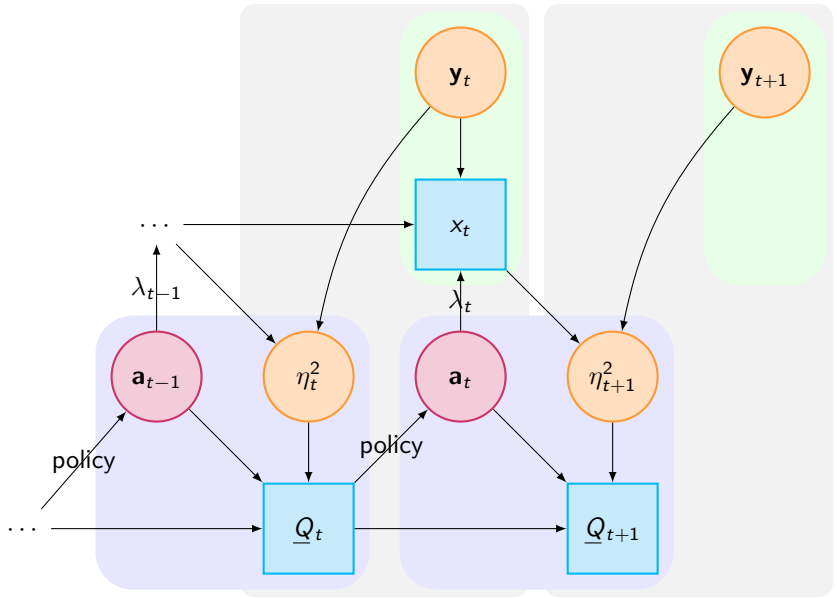

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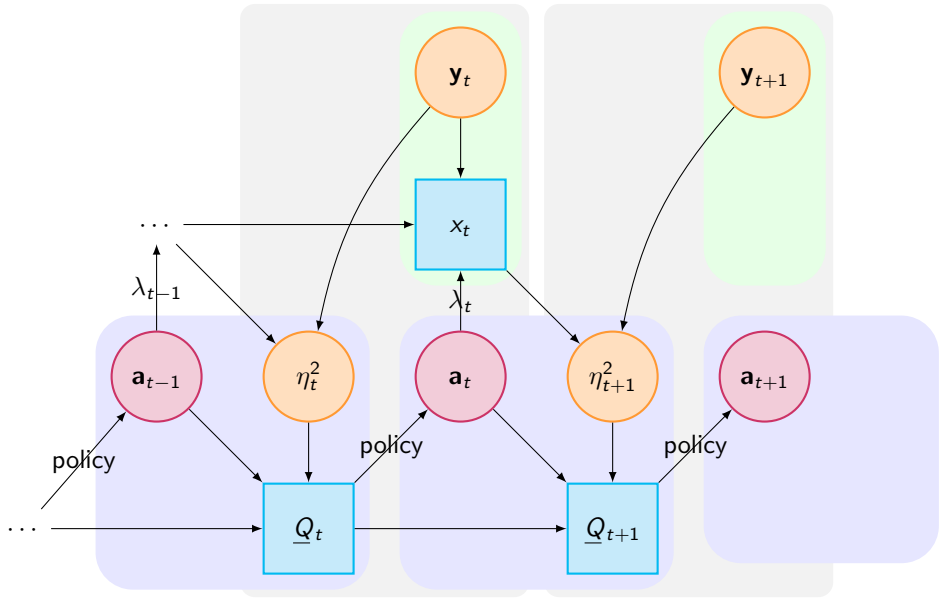

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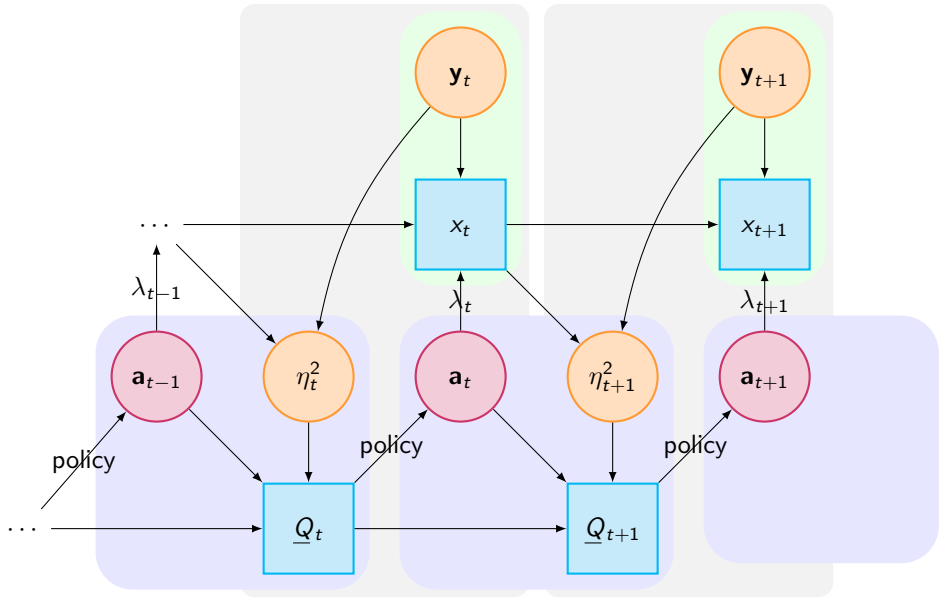

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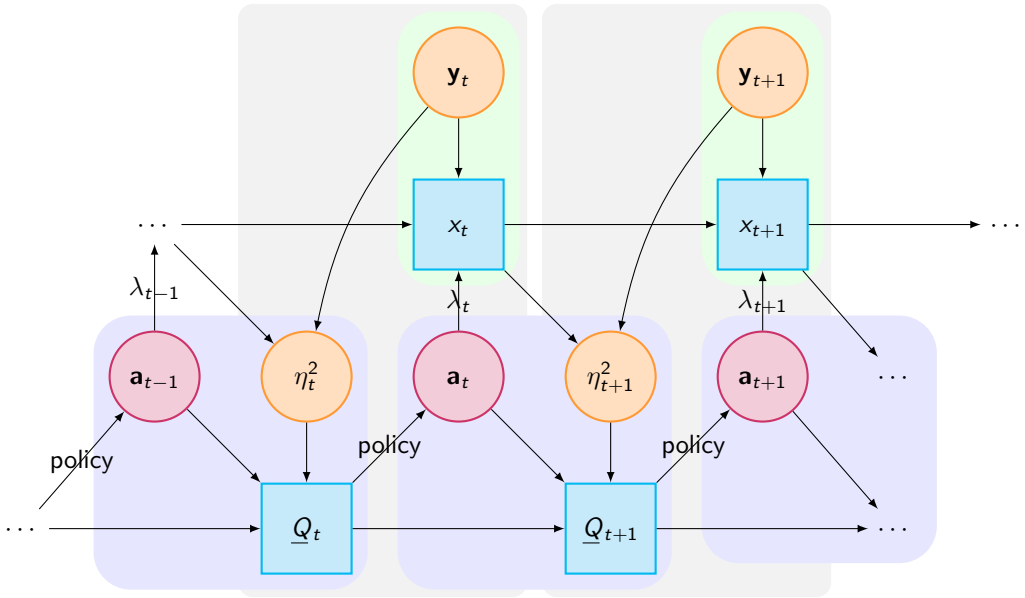

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$$Q_t(a_{t-1}) = Q_{t-1}(a_{t-1}) + \lambda 2\left(\eta_t^2 + \gamma 2 \min \underline{Q}_{t-1} - Q_{t-1}(a_{t-1})\right)$$

a_t It determines its new action by policy: $a_t(\underline{Q}_t)$

u_t It updates its estimate of p:

$$\hat{x}_t = \hat{x}_{t-1} + \lambda_t(a_t) \cdot \eta_t$$

Measure of Quality

	Nr.	Error	Available to Learner	Comment
Estimation	1	$E = \frac{1}{T}\sum_{t=1}^{T} \hat{\mathbf{u}}_t \neq u_t$	no	Reward r maximizer
	2	$E = \frac{1}{T}\sum_{t=1}^{T}(\hat{\mathbf{x}}_t - x_t)^2$	no	
Prediction		$E = \frac{1}{T}\sum_{t=2}^{T} \hat{\mathbf{u}}_t \neq u_{t-1}$	no	
		$E = \frac{1}{T}\sum_{t=2}^{T}(\hat{\mathbf{x}}_t - x_{t-1})^2$	no	
	3	$E = \frac{1}{T}\sum_{t=2}^{T}\underbrace{(y_t - \hat{x}_{t-1})}_{\eta_t}^2$	yes	

Task

Volatility Frequency of change of the better stimuli: $v \in \{0.001, 0.005, 0.01, 0.05\}$

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Run Consist of $T = 5000$ time-steps. Repeated $\longrightarrow$ Statistical results.

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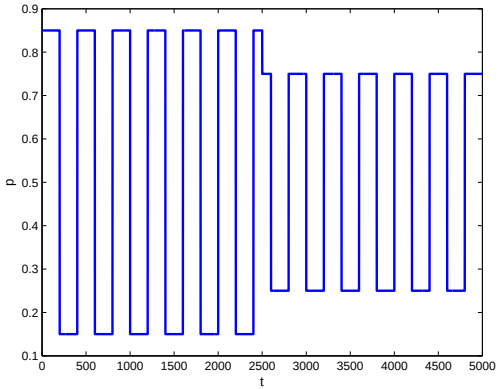

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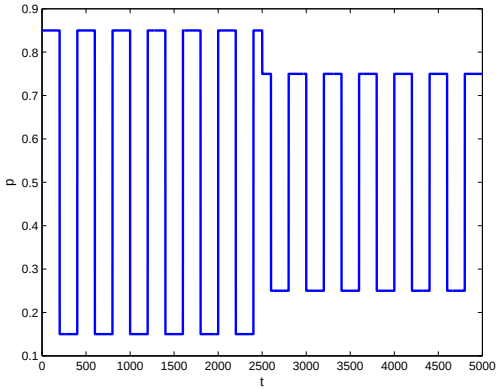

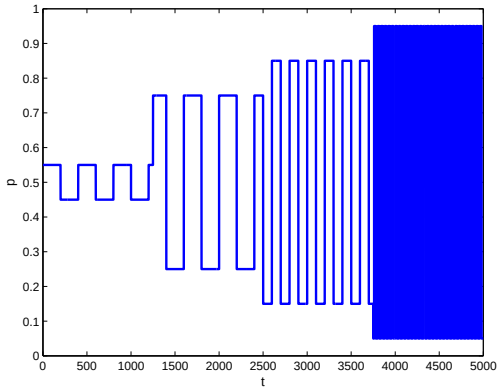

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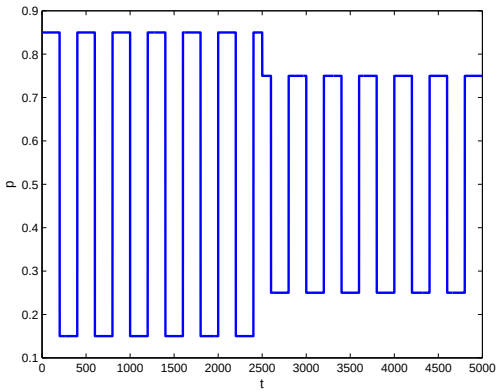

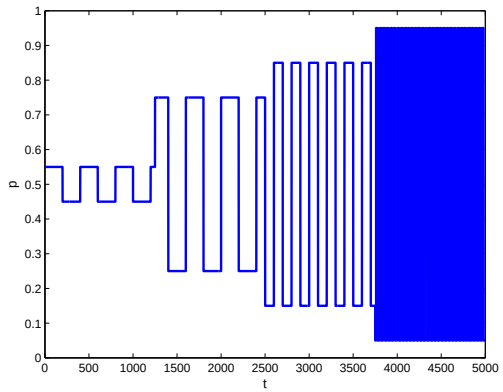

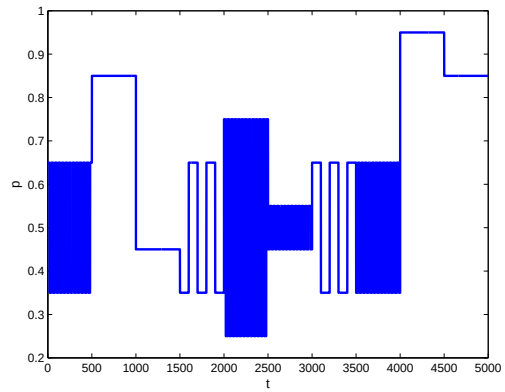

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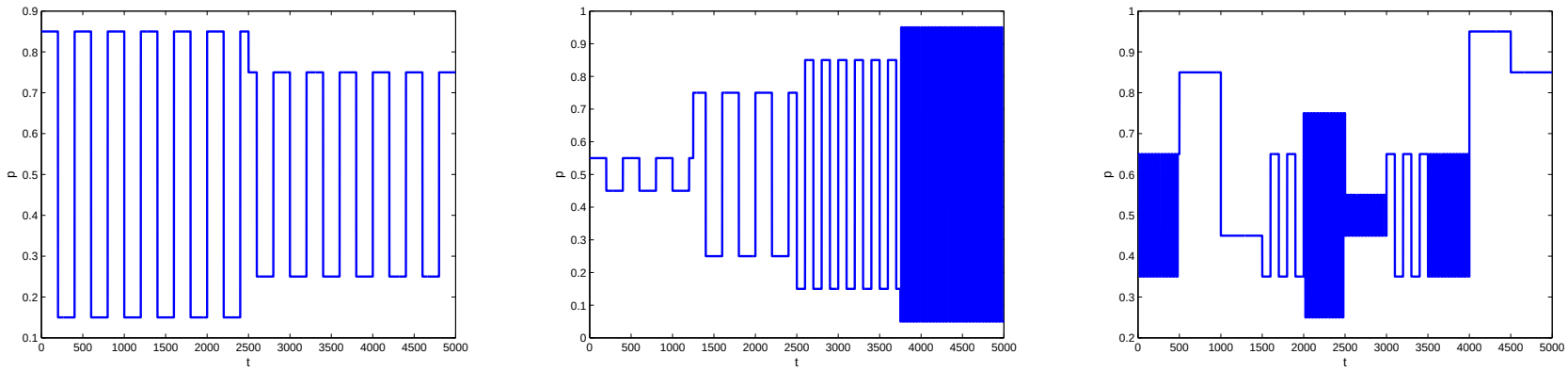
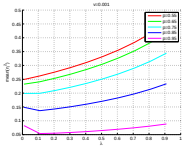


Figure: Example stimuli dynamic task

Fixed Learning Rates

Running learner 1 for fixed learning rates on the static task



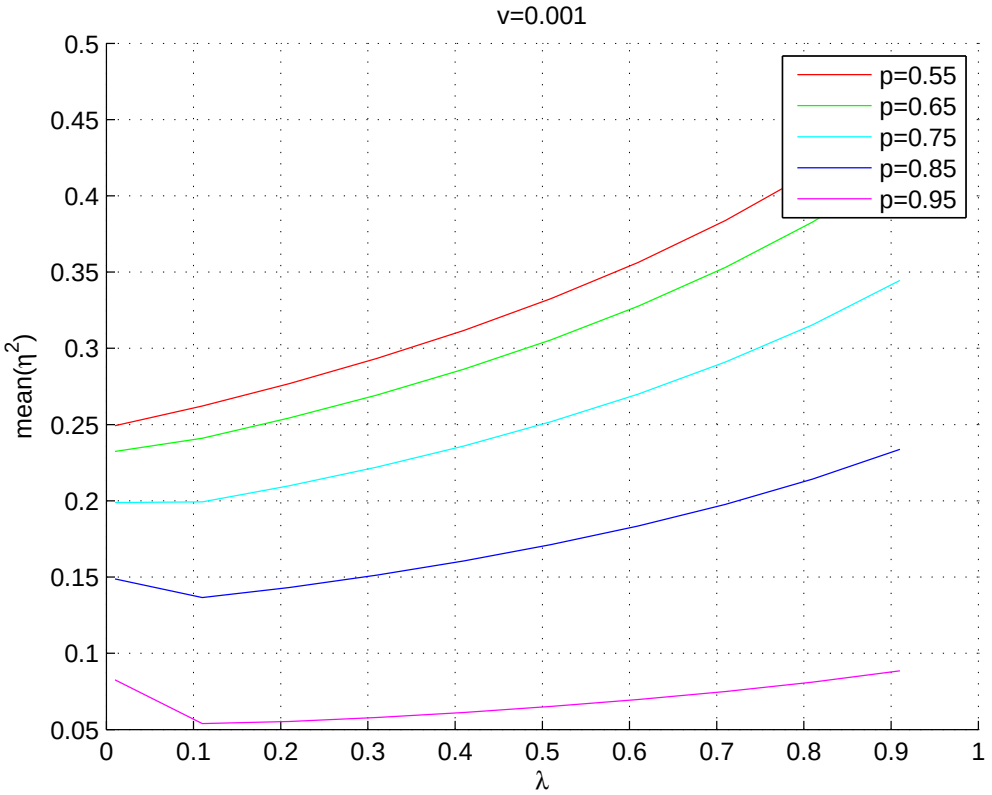


Figure: Error 3 for fixed learning rates

Fixed Learning Rates

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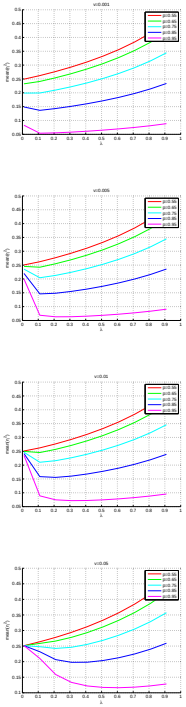

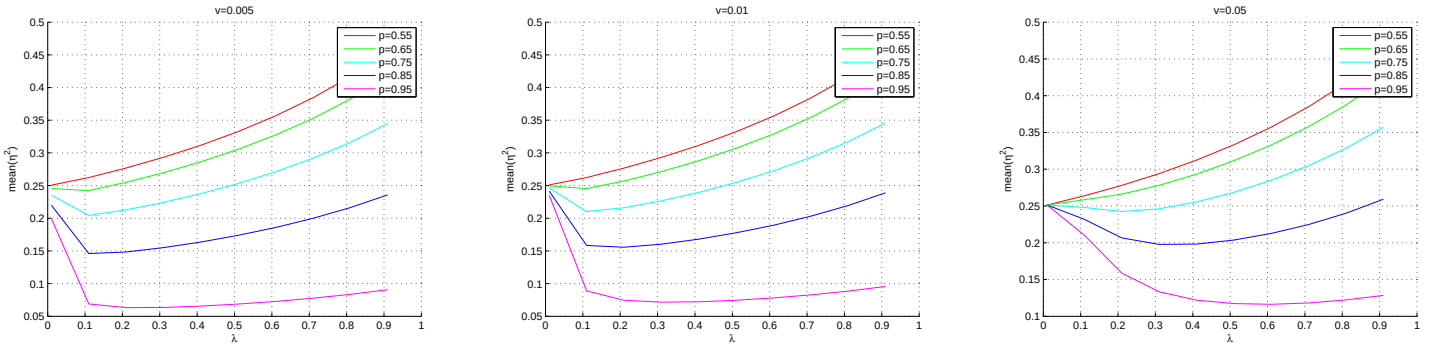

Figure: ... similar for other volatilities

Fixed Learning Rates

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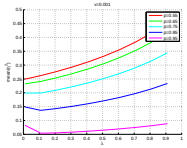
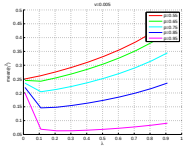
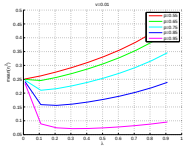
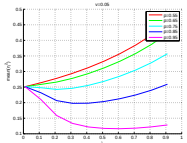
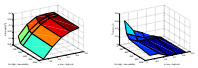
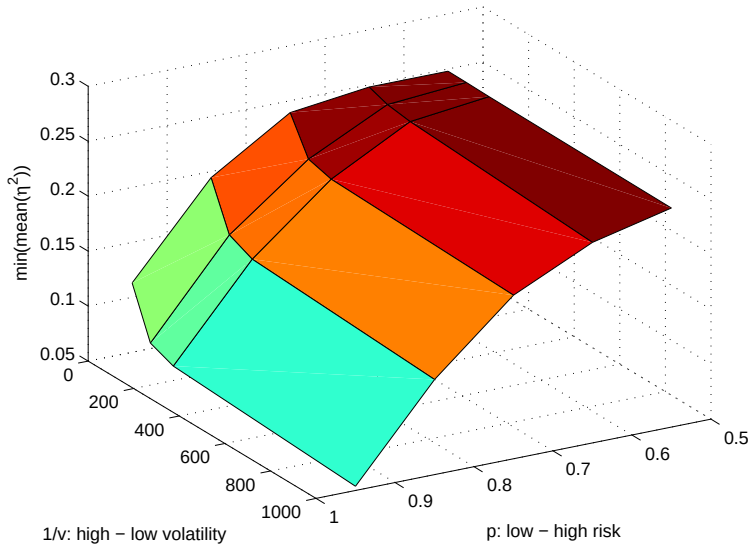




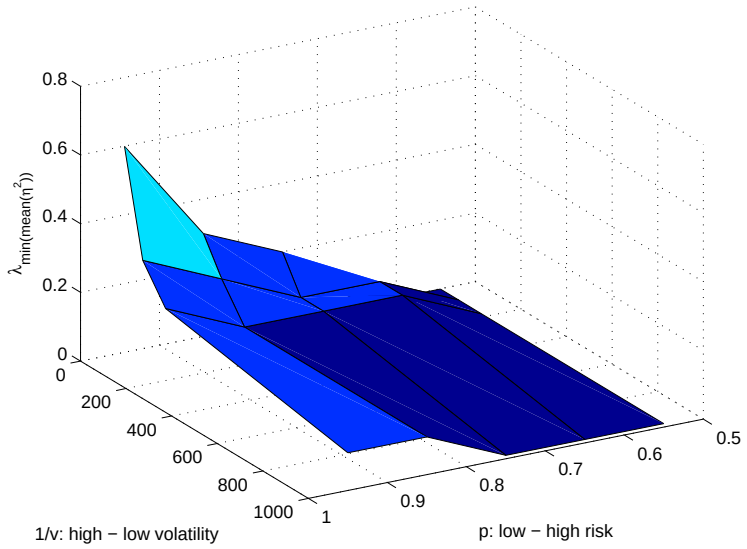








$E_3(\lambda_{opt})$



λ_{opt}

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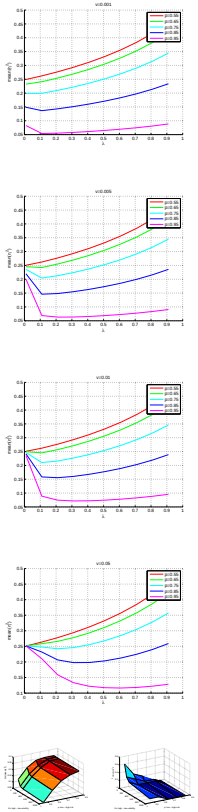

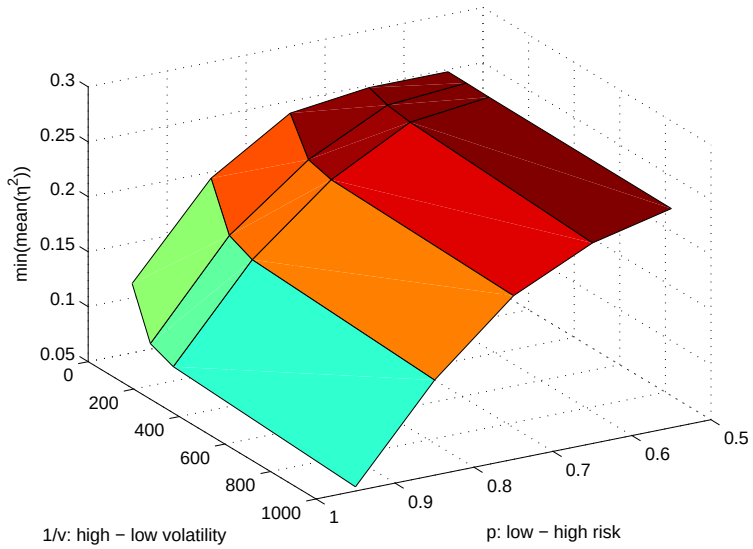

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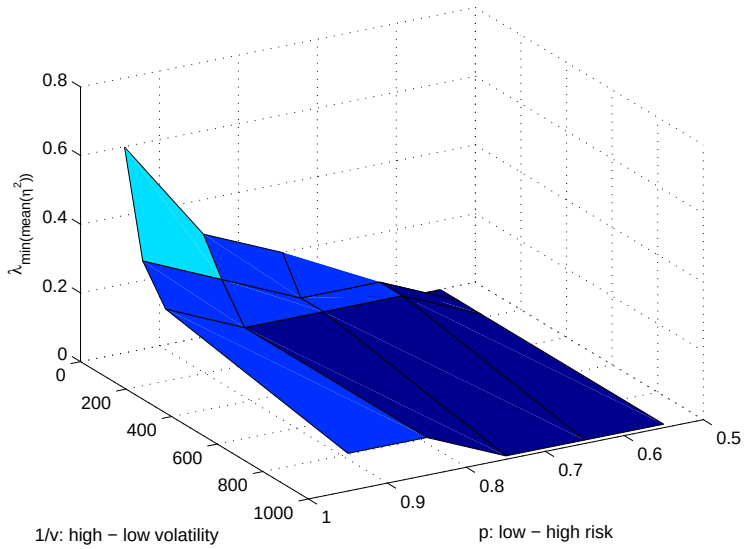

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- Behavior according risk and volatility is given by $\lambda_{opt}(v, p)$

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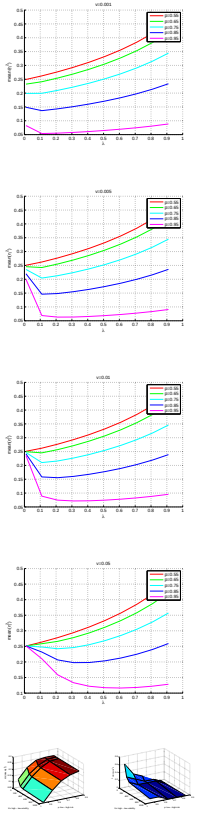

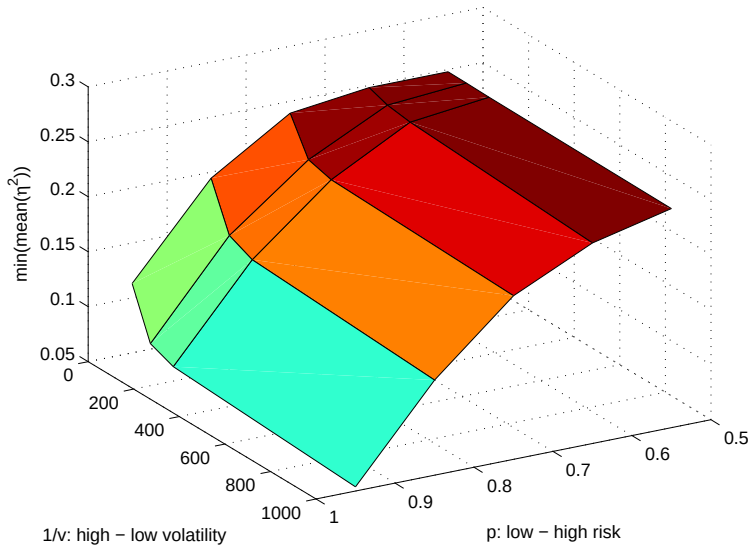

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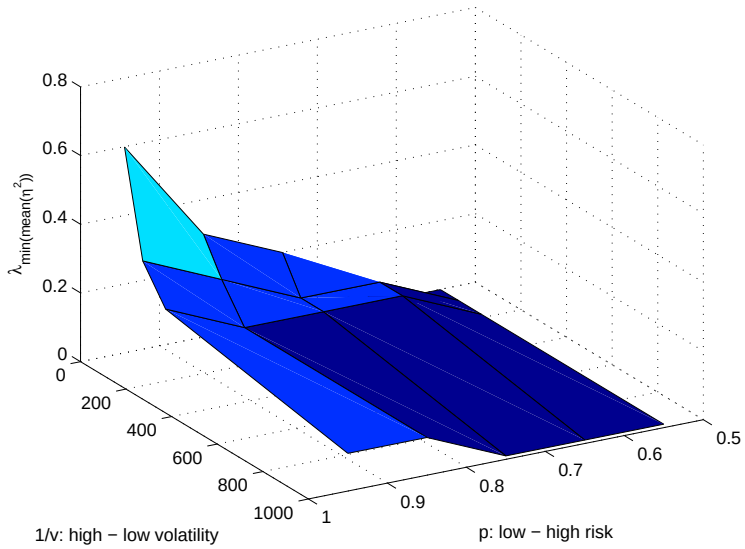

λ_{opt}

- Behavior according risk and volatility is given by $\lambda_{opt}(v, p)$
- mean(η^2) are convex functions
- Get an idea of Q_{init}
- $\lambda_{opt}(v, p)$ is concavel

Testing

Quite a lot of things to test:

Testing

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Initial Values

- Initial Q-Values (Agent 2)

Policy (Agent 2)

- Tabu-Search Policy (permutation, ban)
- Neighborhood Policy (width and shape of neighborhood)
- ϵ-Greedy Policy (ϵ)
- Softmin Policy (τ)

Update (Agent 2)

- Discount rate $\gamma 2 \rightarrow (= 0$: discounted mean$(\eta^2))$
- Learning rate $\lambda 2$
- Neighborhood Q-Update

Others

- Possible learning rates (Agent 1)
- Other methods than two agent setup

Results

Primary

Initial Values $\underline{Q}_{init}$

- ! Need to be above a bound (we do minimization)

Results

Primary

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! Need to be above a bound (we do minimization)

→ We gave estimates for bound

Results

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Results

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Markov Property of 2nd Learner

- Small learning rate (eg. $\lambda_t = 0.01$) $\longrightarrow$ η_t^2 is sparely dependent on λ_t)

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Convexity of $\mathbb{E}\left[\eta^2\right]$

- rc By fixed learning rates experiments:

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Convexity of $\mathbb{E}\left[\eta^2\right]$

- rc By fixed learning rates experiments:
- → One can search within the neighborhood of a learning rate .
- → Helps to overcome the Markov issue.

Results

Secondary

Policy

- Neighborhood: tabu-search and neighborhood search $\gg$ ϵ-greedy and softmin
- Tabu Policy: Results show no benefit from banning

Results

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- Results show no benefit from Q-learning over simple discounted recursive mean learning.
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- Results show no benefit from Q-learning over simple discounted recursive mean learning.
- $\lambda 2$ depend on the dynamics of the task.

Others

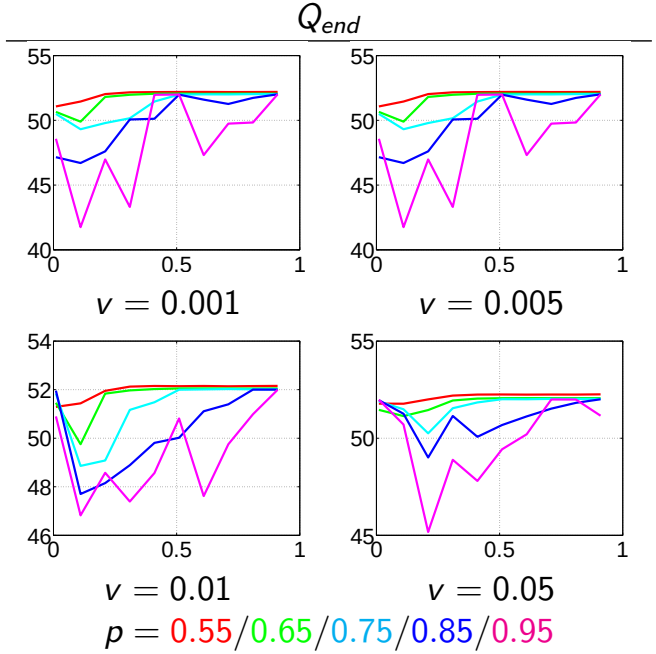
- Better performance with different learning rate set.
- Concurrent learner superior

Results

General Observations on the Task

Tabu Policy

Learner			Results				
	v \ p		0.55	0.65	0.75	0.85	0.95
Q-Learning	λ_{avg}	0.001	0.108	0.117	0.219	0.237	0.313
$Q_{init} = 52$		0.005	0.113	0.104	0.225	0.310	0.290
$\lambda 2 = 0.1$		0.01	0.111	0.117	0.212	0.330	0.296
$\gamma 2 = 0.995$		0.05	0.141	0.143	0.236	0.373	0.323
	η^2_{avg}	0.001	0.269	0.244	0.214	0.154	0.062
Tabu-Policy		0.005	0.271	0.249	0.216	0.169	0.085
boundary $= 8$		0.01	0.270	0.250	0.220	0.168	0.116
$\sigma^2 = 0.25^2$		0.05	0.278	0.266	0.246	0.210	0.148
	$\lvert x - \hat{x} \rvert_{avg}$	0.001	0.111	0.097	0.124	0.107	0.066
		0.005	0.119	0.116	0.130	0.152	0.108
		0.01	0.119	0.117	0.138	0.138	0.161
		0.05	0.137	0.152	0.172	0.183	0.161
	right est.	0.001	68%	88%	92%	95%	98%
		0.005	62%	80%	91%	90%	94%
		0.01	60%	79%	89%	92%	88%
		0.05	54%	64%	79%	85%	90%

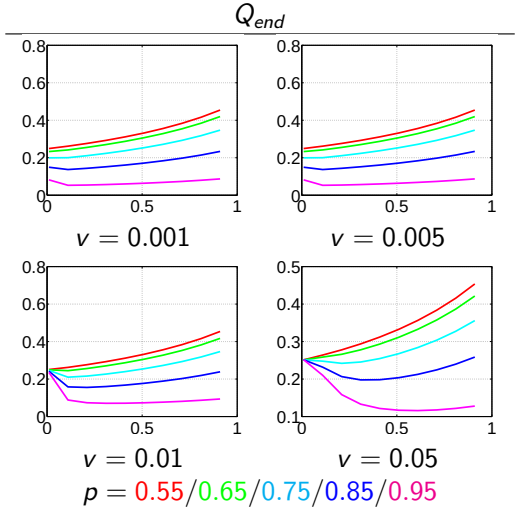


Results

General Observations on the Task

Tabu Policy | Concurrent Learner

Learner	Results						
Concurrent Learning $\lambda 2 = \frac{1}{t}$ Policy $\underset{a}{\text{argmin}}\ \eta^2$	v \ p		0.55	0.65	0.75	0.85	0.95
	λ_{avg}	0.001	0.011	0.013	0.050	0.107	0.118
		0.005	0.011	0.097	0.110	0.113	0.247
		0.01	0.011	0.102	0.111	0.208	0.329
		0.05	0.011	0.012	0.211	0.346	0.603
	η^2_{avg}	0.001	0.249	0.233	0.199	0.137	0.053
		0.005	0.250	0.242	0.205	0.146	0.062
		0.01	0.251	0.244	0.210	0.155	0.071
		0.05	0.251	0.251	0.241	0.197	0.116
	$\lvert x - \hat{x} \rvert_{avg}$	0.001	0.033	0.048	0.068	0.071	0.048
		0.005	0.047	0.101	0.095	0.089	0.070
		0.01	0.050	0.106	0.106	0.115	0.079
		0.05	0.052	0.149	0.161	0.162	0.095
	right est.*	0.001	85%	94%	95%	99%	100%
		0.005	62%	85%	96%	97%	99%
		0.01	56%	82%	93%	96%	98%
		0.05	51%	52%	80%	89%	95%

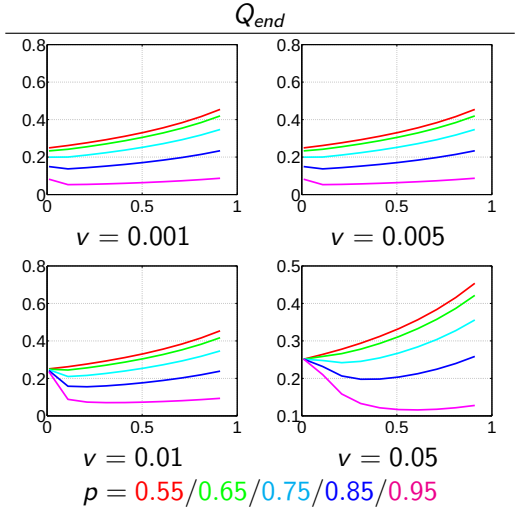


Results

General Observations on the Task

Tabu Policy | Concurrent Learner | Comparison

Learner			Results				
Concurrent Learning $\lambda 2 = \frac{1}{t}$ Policy $\underset{a}{\text{argmin}}\ \eta^2$	v \ p		0.55	0.65	0.75	0.85	0.95
	λ_{avg}	0.001	0.011	0.013	0.050	0.107	0.118
		0.005	0.011	0.097	0.110	0.113	0.247
		0.01	0.011	0.102	0.111	0.208	0.329
		0.05	0.011	0.012	0.211	0.346	0.603
	η^2_{avg}	0.001	0.249 (93%)	0.233 (95%)	0.199 (93%)	0.137 (89%)	0.053 (85%)
		0.005	0.250 (92%)	0.242 (97%)	0.205 (95%)	0.146 (86%)	0.062 (73%)
		0.01	0.251 (93%)	0.244 (98%)	0.210 (95%)	0.155 (92%)	0.071 (61%)
		0.05	0.251 (90%)	0.251 (94%)	0.241 (98%)	0.197 (94%)	0.116 (78%)
	$\lvert x - \hat{x} \rvert_{avg}$	0.001	0.033 (30%)	0.048 (49%)	0.068 (55%)	0.071 (66%)	0.048 (72%)
		0.005	0.047 (39%)	0.101 (87%)	0.095 (73%)	0.089 (58%)	0.070 (64%)
		0.01	0.050 (42%)	0.106 (90%)	0.106 (77%)	0.115 (83%)	0.079 (49%)
		0.05	0.052 (38%)	0.149 (98%)	0.161 (93%)	0.162 (89%)	0.095 (59%)
	right est.*	0.001	85% (428%)	94% (143%)	95% (121%)	99% (130%)	100% (113%)
		0.005	62% (92%)	85% (144%)	96% (128%)	97% (149%)	99% (134%)
		0.01	56% (39%)	82% (122%)	93% (125%)	96% (128%)	98% (184%)
		0.05	51% (2%)	52% (2%)	80% (112%)	89% (130%)	95% (137%)



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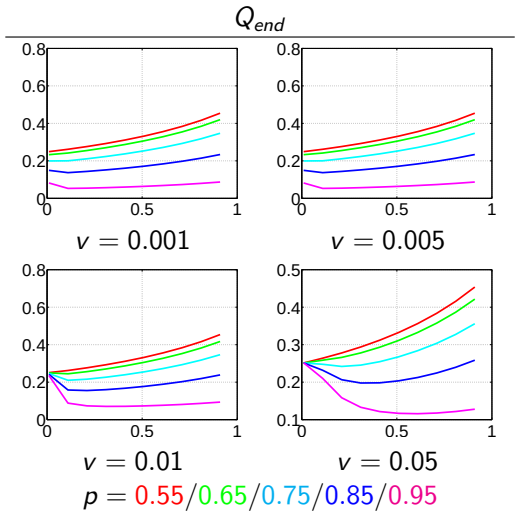

Minimizing $\eta^2 \not\Rightarrow$ Better estimation of the token = Higher Reward

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Results

General Observations on the Task

Tabu Policy | Concurrent Learner | Comparison | Consequences

Different λ_{opt}

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Experiment

This and also the fact that a simple concurrent learner performs significant better demands for thinking about

- The purpose of the experiment
- The experiment itself

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How do human learners treat risk and volatility?

Does reinforcement learning benefit from Q-learning the learning rate?

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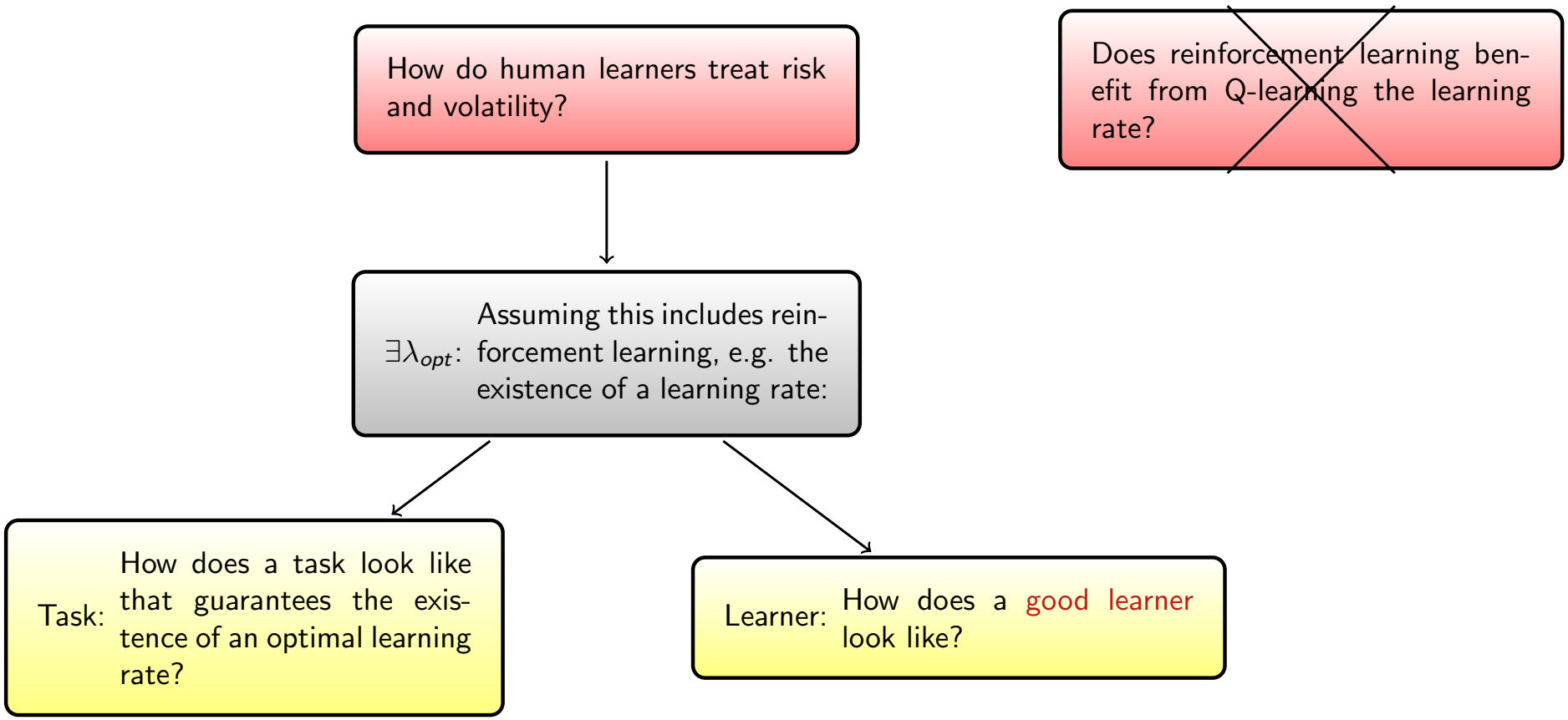
How do human learners treat risk and volatility?
Does reinforcement learning benefit from Q-learning the learning rate?
$\exists\lambda_{opt}$: Assuming this includes reinforcement learning, e.g. the existence of a learning rate:
Task: How does a task look like that guarantees the existence of an optimal learning rate?
Learner: How does a good learner look like?

Task, Realization & Learning

Example Task

Given	hidden truth x
Observe	noisy $\mathbf{y}$
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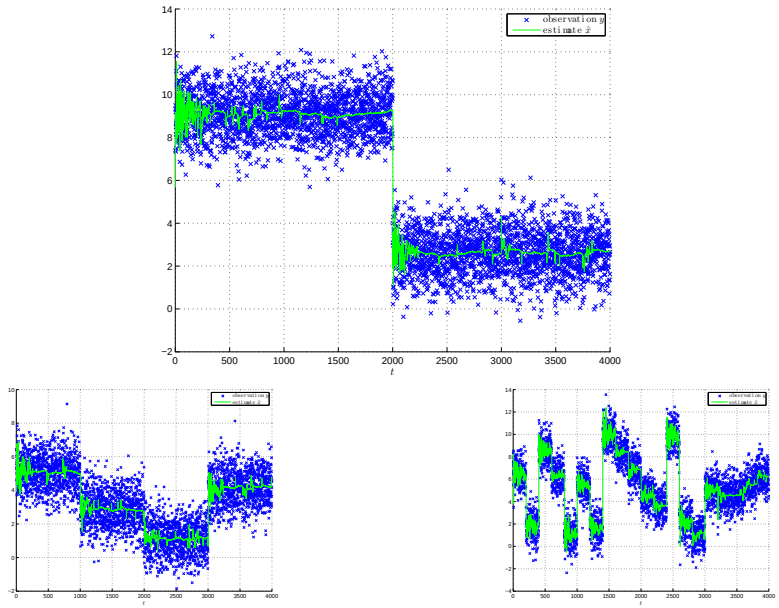

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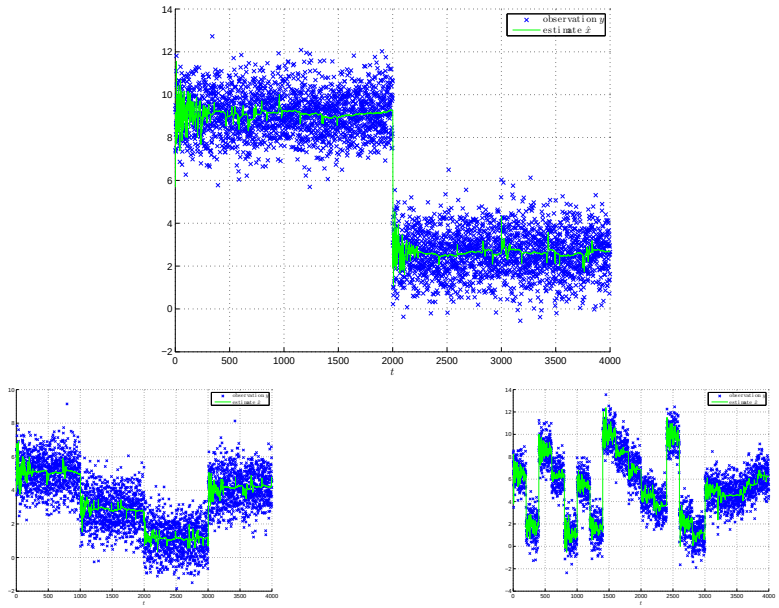

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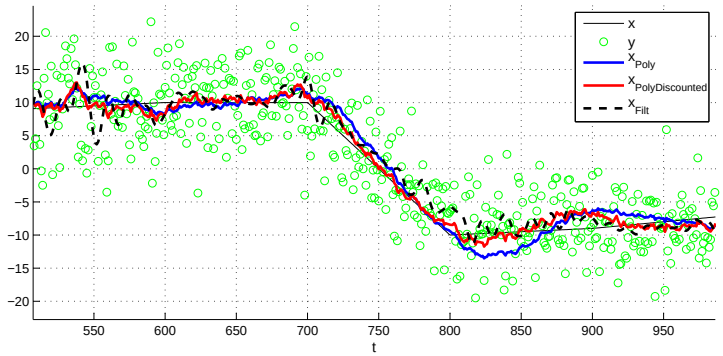
Further: Slopes



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Task, Realization & Learning

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Task, Realization & Learning

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Further: Recursive parabola matching, multiple states Q-Learning

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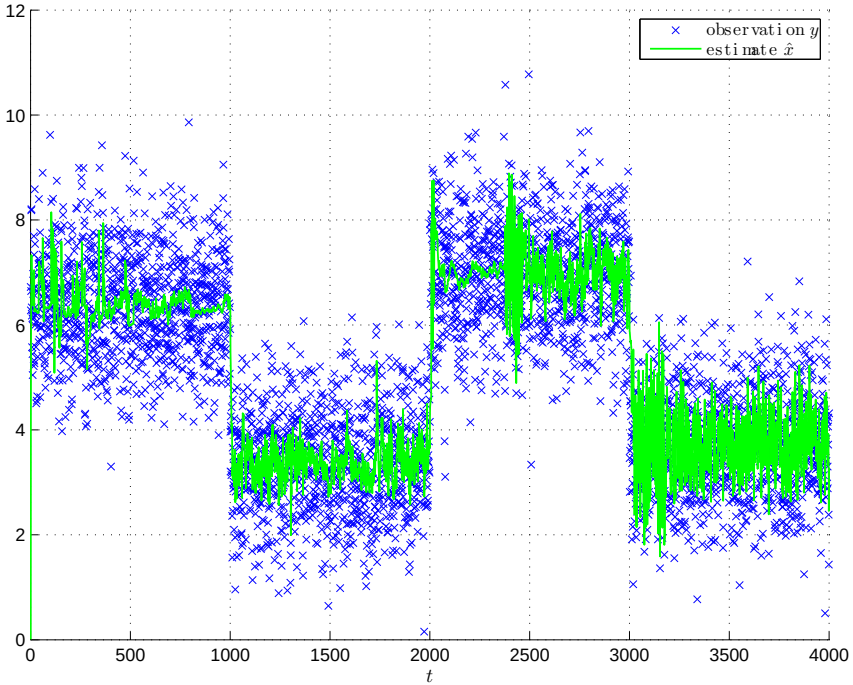

Tabu Learner

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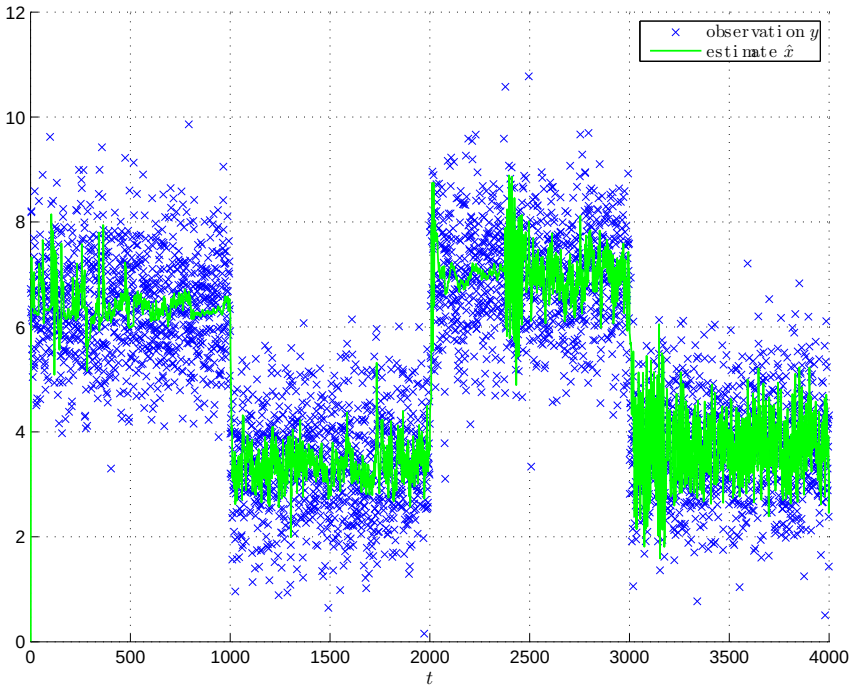
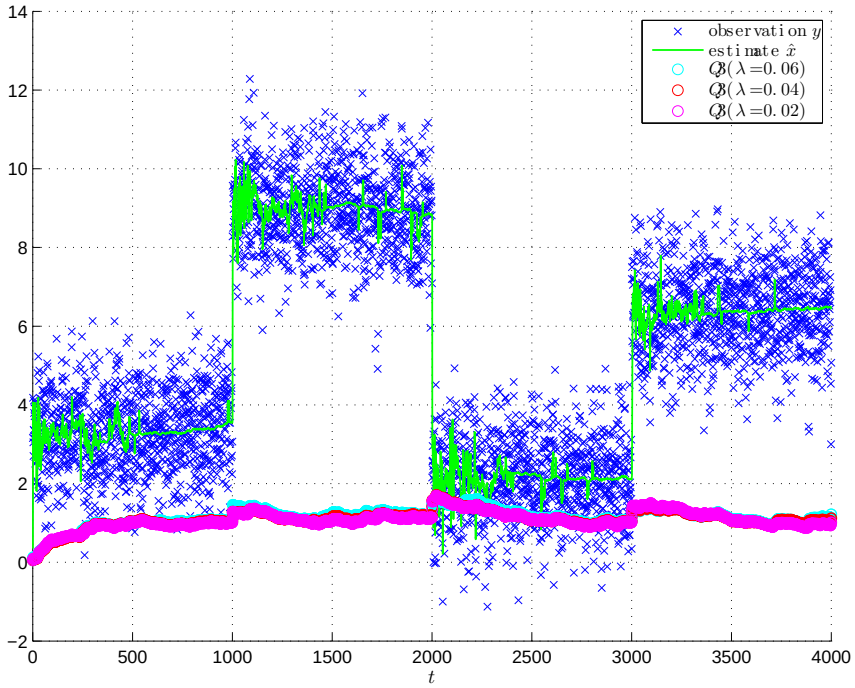
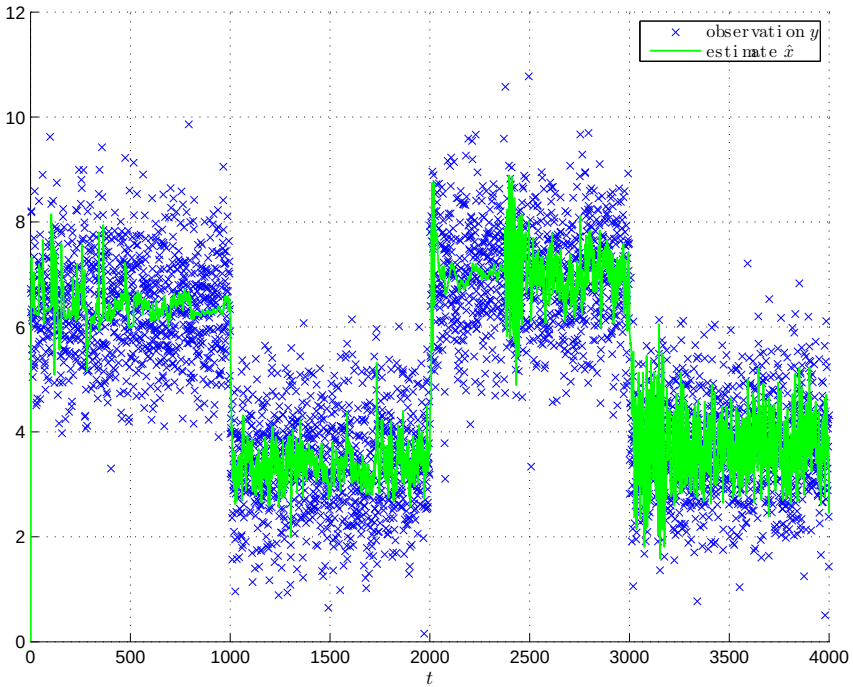


Tabu Learner



3-Layer Learner

Example Run



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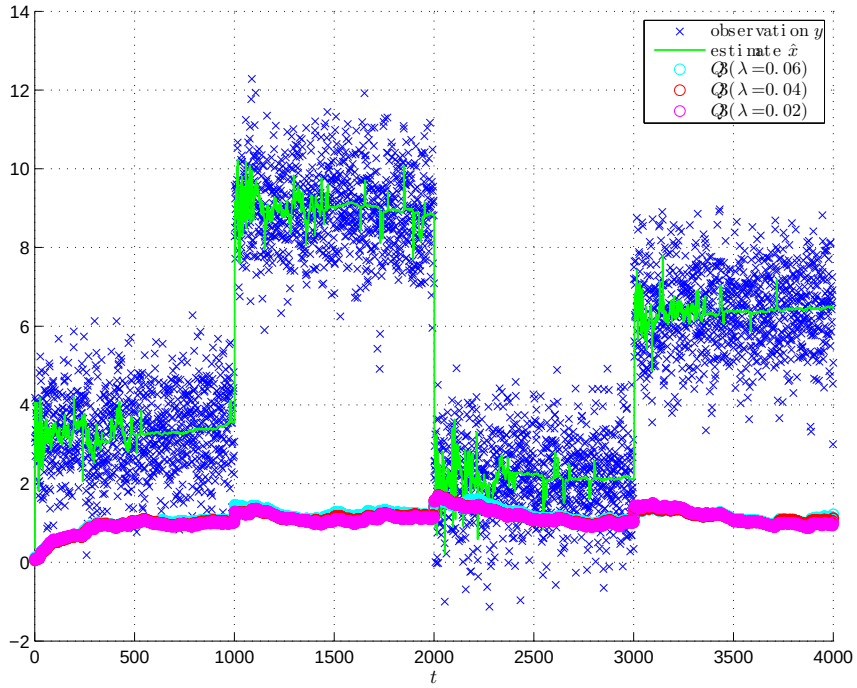

3-Layer Learner

Method	Parameters		mean$(x-\hat{x})^2$		
	$\lambda 2$	$\lambda 3$	C=2	C=4	C=20
Tabu policy single learner			0.132	0.182	0.607
2-Layer	0.06		0.067 (50.7%)	0.084 (46.3%)	0.203 (33.5%)
	0.04		0.053 (40.5%)	0.074 (40.8%)	0.201 (**33.1%**)
	0.02		0.044 (**33.5%**)	0.069 (**37.9%**)	0.218 (36.0%)
3-Layer	0.02, 0.04, 0.06	0.005	0.044 (33.1%)	0.071 (38.9%)	0.204 (33.6%)

The three layer method combines the good performance of different $\lambda 2$ learners.

Conclusion and Outlook ($\Rightarrow$)

For using PRL to compare machine to human according to risk and volatility

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+ Task: λ_{opt}(Volatility, Risk) (opposite as proposed in paper)

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Q-Learning the Learning Rate

⇒ We did not focus further on this Idea. But we assume potential when using in a multi-state setup.

Questions?